① Resolva os seguintes sistemas pelo método da <u>adição ordenada</u>

a) $\begin{cases} x_1 + x_2 + 2x_3 + x_4 = 5 \\ 2x_1 + 3x_2 - x_3 - 2x_4 = 2 \\ 4x_1 + 5x_2 + 3x_3 = 7 \end{cases} \underset{L_3 - 4L_1}{\overset{L_2 - 2L_1}{\Leftrightarrow}} \begin{cases} x_1 + x_2 + 2x_3 + x_4 = 5 \\ x_2 - 5x_3 - 4x_4 = -8 \\ x_2 - 5x_3 - 4x_4 = -13 \end{cases} \Leftrightarrow$

$\overset{L_3 - L_2}{\Leftrightarrow} \begin{cases} \\ \\ 0 = -5 \end{cases}$ sistema impossível

b) sistema homogéneo

$\begin{cases} x_1 - x_2 + x_3 = 0 \\ x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + x_2 + 3x_3 = 0 \end{cases} \overset{L_3 - 2L_1}{\Leftrightarrow} \begin{cases} x_1 - x_2 + x_3 = 0 \\ 3x_2 - 2x_3 = 0 \\ 3x_2 + x_3 = 0 \end{cases} \overset{L_3 - L_2}{\Leftrightarrow} \begin{cases} x_1 - x_2 + x_3 = 0 \\ 3x_2 - 2x_3 = 0 \\ 3x_3 = 0 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases} \Leftrightarrow (x_1, x_2, x_3) = (0, 0, 0)$ sistema possível determinado

c) $\begin{cases} x_1 + x_2 - x_3 + x_4 = 2 \\ 2x_1 - x_2 + x_3 - 3x_4 = 1 \end{cases} \overset{L_2 - 2L_1}{\Leftrightarrow} \begin{cases} x_1 + x_2 - x_3 + x_4 = 2 \\ -3x_2 + 3x_3 - 5x_4 = -3 \end{cases} \Leftrightarrow \begin{cases} x_1 + \frac{3 + 3x_3 - 5x_4}{3} - x_3 + x_4 = 2 \\ x_2 = \frac{3 + 3x_3 - 5x_4}{3} \end{cases}$

$\Leftrightarrow \begin{cases} x_1 + 1 + x_3 - \frac{5x_4}{3} - x_3 + x_4 = 2 \\ \text{—} \end{cases} \Leftrightarrow \begin{cases} -5x_4 = 1 - x_1 - x_4 + 3 \\ \text{—} \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 5x_4 = -3 + 3x_1 + 3x_4 \\ \text{—} \end{cases} \Leftrightarrow \begin{cases} 5x_4 - 3x_4 = -3 + 3x_1 \\ \text{—} \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x_4 = \frac{3x_1 - 3}{2} \\ -3x_2 + 3x_3 - \frac{3x_1 - 3}{2} \cdot 5 = -3 \end{cases} \Leftrightarrow \begin{cases} x_4 = \frac{3x_1 - 3}{2} \\ x_2 = \frac{3 + 3x_3 - 5x_4}{3} \end{cases} \Leftrightarrow \begin{cases} x_4 = \frac{3x_1}{2} - \frac{3}{2} \\ x_2 = 1 + x_3 - \frac{5x_4}{3} \end{cases}$

$\Leftrightarrow (x_1, x_2, x_3, x_4) = \left(x_1, 1 + x_3 - \frac{5x_4}{3}, x_3, \frac{3x_1}{2} - \frac{3}{2}\right), \forall x_3, x_4 \in \mathbb{R}$ SPI

d) $\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_1 + 3x_2 + 2x_3 + 4x_4 = 0 \\ 2x_1 + x_2 - x_4 = 0 \end{cases} \underset{L_2 - L_1}{\overset{L_3 - 2L_1}{\Leftrightarrow}} \begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ 2x_2 + x_3 + 3x_4 = 0 \\ -x_2 - 2x_3 - 3x_4 = 0 \end{cases} \overset{L_3 + \frac{1}{2}L_2}{\Leftrightarrow} \begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ 2x_2 + x_3 + 3x_4 = 0 \\ -\frac{3}{2}x_3 - \frac{3}{2}x_4 = 0 \end{cases}$

$\Leftrightarrow \begin{cases} \text{—} \\ \text{—} \\ -\frac{3}{2}(x_3 + x_4) = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 + x_2 - x_4 + x_4 = 0 \\ 2x_2 - x_4 + 3x_4 = 0 \\ x_3 = -x_4 \end{cases} \Leftrightarrow \begin{cases} \\ 2x_2 + 2x_4 = 0 \\ \\ \end{cases} \Leftrightarrow$

$$\Leftrightarrow \begin{cases} x_1 = -x_2 \\ x_2 = -x_4 \\ x_3 = -x_4 \end{cases} \Leftrightarrow \begin{cases} x_1 = x_4 \\ x_2 = -x_4 \\ x_3 = -x_4 \end{cases} \Leftrightarrow (x_1, x_2, x_3, x_4) = (x_4, -x_4, -x_4, x_4),$$

$\forall x_4 \in \mathbb{R}$

SPI

e) $\begin{cases} x_1 - x_2 + 2x_3 + x_4 = 1 \\ 2x_1 + x_2 - x_3 + 3x_4 = 3 \\ x_1 + 5x_2 - 8x_3 + x_4 = 1 \\ 4x_1 + 5x_2 - 7x_3 + 7x_4 = 7 \end{cases}$ $\underset{\substack{L_3 - L_1 \\ L_2 - 2L_1}}{\overset{L_4 - 4L_1}{\Leftrightarrow}}$ $\begin{cases} x_1 - x_2 + 2x_3 + x_4 = 1 \\ 3x_2 - 5x_3 + x_4 = 1 \\ 6x_2 - 10x_3 = 0 \\ 9x_2 - 15x_3 + 3x_4 = 3 \end{cases}$ $\Leftrightarrow$

$\underset{L_3 - 2L_2}{\overset{L_4 - 3L_2}{\Leftrightarrow}}$ $\begin{cases} x_1 - x_2 + 2x_3 + x_4 = 1 \\ 3x_2 - 5x_3 + x_4 = 1 \\ 0 = 2x_4 = -2 \\ 0 = 0 \end{cases}$ $\Leftrightarrow$ $\begin{cases} \\ 3x_2 - 5x_3 + 1 = 1 \\ x_4 = 1 \\ 0 = 0 \end{cases}$ $\Leftrightarrow$ $\begin{cases} x_1 - \frac{5x_3}{3} + \frac{6x_3}{3} = 0 \\ x_2 = \frac{5x_3}{3} \\ x_4 = 1 \end{cases}$ $\Leftrightarrow$

$\Leftrightarrow \begin{cases} x_1 + \frac{x_3}{3} = 0 \\ \\ \end{cases} \Leftrightarrow \begin{cases} x_1 = -\frac{x_3}{3} \\ x_2 = \frac{5x_3}{3} \end{cases} \Leftrightarrow (x_1, x_2, x_3, x_4) = \left(-\frac{x_3}{3}, \frac{5x_3}{3}, x_3, 1\right), \forall x_3 \in \mathbb{R}$

Sistema impossível indeterminado (de grau 1)

② Resolva os seguintes sistemas de equações lineares (SEL) pelo método de eliminação de Gauss

a) $\begin{cases} x_1 + x_2 + 2x_3 + x_4 = 5 \\ 2x_1 + 3x_2 - x_3 - 2x_4 = 2 \\ 4x_1 + 5x_2 + 3x_3 = 7 \end{cases}$ $\Leftrightarrow$ $\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 5 \\ 2 & 3 & -1 & -2 & 2 \\ 4 & 5 & 3 & 0 & 7 \end{array}\right]$ $\underset{\Leftrightarrow}{C_1 \leftrightarrow C_4}$ $\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 5 \\ -2 & 3 & -1 & 2 & 2 \\ 0 & 5 & 3 & 4 & 7 \end{array}\right]$ $\Leftrightarrow$

$\underset{\Leftrightarrow}{L_2 + 2L_1}$ $\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 5 \\ 0 & 5 & 3 & 4 & 12 \\ 0 & 5 & 3 & 4 & 7 \end{array}\right]$ $\underset{\Leftrightarrow}{L_3 - L_2}$ $\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 5 \\ 0 & 5 & 3 & 4 & 12 \\ 0 & 0 & 0 & 0 & -5 \end{array}\right] \rightarrow$ SI

b) $\begin{cases} x_1 - x_2 + x_3 = 0 \\ x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + x_2 + 3x_3 = 0 \end{cases}$ $\Leftrightarrow$ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 1 & 2 & -1 & 0 \\ 2 & 1 & 3 & 0 \end{array}\right]$ $\underset{\substack{\Leftrightarrow \\ L_3 - 2L_1}}{L_2 - L_1}$ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -2 & 0 \\ 0 & 3 & 1 & 0 \end{array}\right]$ $\underset{\Leftrightarrow}{L_3 - L_2}$ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -2 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right]$

$\Leftrightarrow \begin{cases} x_1 - x_2 + x_3 = 0 \\ 3x_2 - 2x_3 = 0 \\ 3x_3 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases} \Leftrightarrow (x_1, x_2, x_3) = (0, 0, 0)$

c) $\begin{cases} x_1 - x_2 + 2x_3 + x_4 = 1 \\ 2x_1 + x_2 - x_3 + 3x_4 = 3 \\ x_1 + 5x_2 - 8x_3 + x_4 = 1 \\ 4x_1 + 5x_2 - 7x_3 + 7x_4 = 7 \end{cases} \Leftrightarrow \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 2 & 1 & -1 & 3 & 3 \\ 1 & 5 & -8 & 1 & 1 \\ 4 & 5 & -7 & 7 & 7 \end{array}\right] \begin{array}{c} L_4 - 4L_1 \\ L_3 - L_1 \\ \Leftrightarrow \\ L_2 - 2L_1 \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 3 & -5 & 1 & 1 \\ 0 & 6 & -10 & 0 & 0 \\ 0 & 9 & -15 & 3 & 3 \end{array}\right] \Leftrightarrow$

$\begin{array}{c} L_4 - 3L_2 \\ \Leftrightarrow \\ L_3 - 2L_2 \\ \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 3 & -5 & 1 & 1 \\ 0 & 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Leftrightarrow \begin{cases} x_1 - x_2 + 2x_3 + x_4 = 1 \\ 3x_2 - 5x_3 + x_4 = 1 \\ -2x_4 = -2 \\ 0 = 0 \end{cases} \Leftrightarrow \begin{cases} 3x_2 = 5x_3 \\ x_4 = 1 \\ 0 = 0 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x_1 - \frac{5x_3}{3} + 2x_3 = 0 \\ x_2 = \frac{5x_3}{3} \\ x_4 = 1 \end{cases} \Leftrightarrow \begin{cases} x_1 = \frac{-x_3}{3} \\ \text{—} \\ \text{—} \end{cases} \Leftrightarrow (x_1, x_2, x_3, x_4) = \left(\frac{-x_3}{3}, \frac{5x_3}{3}, x_3, 1\right)$, $x_3 \in \mathbb{R}$

SPI

Propostos

1) classifique os seguintes SEL

c) $\begin{cases} 2x - 3y + z = 1 \\ -2x + 3y - z = 2 \\ -4x - 5y + 2z = -4 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} 2 & -3 & 1 & 1 \\ -2 & 3 & -1 & 2 \\ -4 & -5 & 2 & -4 \end{array}\right] \begin{array}{c} L_3 + L_2 \\ \Leftrightarrow \\ L_2 + L_1 \end{array} \left[\begin{array}{ccc|c} 2 & -3 & 1 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & -8 & 3 & -3 \end{array}\right]$ impossível

d) $\begin{cases} x + 3y - 2z - w = -1 \\ -6x - 15y + 9z + 9w = 9 \\ -x - z + 4w = 4 \\ 4x + 10y - 5z - 2w = -5 \end{cases} \Leftrightarrow \left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & -1 \\ -6 & -15 & 9 & 9 & 9 \\ -1 & 0 & -1 & 4 & 4 \\ 4 & 10 & -5 & -2 & -5 \end{array}\right] \begin{array}{c} L_4 - 4L_1 \\ L_3 + L_1 \\ \Leftrightarrow \\ L_2 + 6L_1 \end{array} \left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & -1 \\ 0 & 3 & -3 & 3 & 3 \\ 0 & 3 & -3 & 3 & 3 \\ 0 & -2 & 3 & 2 & -1 \end{array}\right]$

$\begin{array}{c} 3L_4 + 2L_2 \\ \Leftrightarrow \\ L_3 - L_2 \\ \end{array} \left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & -1 \\ 0 & 3 & -3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 12 & 3 \end{array}\right] \Leftrightarrow \begin{cases} x + 3y - 2z - w = -1 \\ 3y - 3z + 3w = 3 \\ 0 = 0 \\ 3z + 12w = 3 \end{cases} \Leftrightarrow \begin{cases} x + 3y - 2z - w = -1 \\ 3y - 3 + 12w + 3w = 3 \\ 0 = 0 \\ z = 1 - 4w \end{cases}$

$\Leftrightarrow \begin{cases} x + 6 - 15w - 2 + 8w - w = -1 \\ 3y = 2 - 5w \\ \text{—} \end{cases} \Leftrightarrow \begin{cases} x = -5 + 8w \\ y = 2 - 5w \\ z = 1 - 4w \end{cases} \Leftrightarrow (x, y, z, w) = (-5 + 8w, 2 - 5w, 1 - 4w, w)$, $w \in \mathbb{R}$

SPI → de grau 1

③ Determine para que valores de $\underline{a} \in \mathbb{R}$ o sistema: $\begin{cases} 2x+2y-4z=0 \\ x-2y+4z=0 \\ 2x+y+az=0 \end{cases}$ admite unicamente a solução nula?

$$\begin{cases} 2x+2y-4z=0 \\ x-2y+4z=0 \\ 2x+y+az=0 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 2 & 2 & -4 & 0 \\ 1 & -2 & 4 & 0 \\ 2 & 1 & a & 0 \end{array}\right] \underset{\Rightarrow}{L_3 \leftrightarrow L_1} \left[\begin{array}{ccc|c} 2 & 1 & a & 0 \\ 1 & -2 & 4 & 0 \\ 2 & 2 & -4 & 0 \end{array}\right] \Rightarrow$$

$$\begin{array}{c} L_3-L_1 \\ \Leftrightarrow \\ 2L_2-L_1 \end{array} \left[\begin{array}{ccc|c} 2 & 1 & a & 0 \\ 0 & -5 & 8-a & 0 \\ 0 & 1 & -4-a & 0 \end{array}\right] \underset{\Rightarrow}{5L_3+L_2} \left[\begin{array}{ccc|c} 2 & 1 & a & 0 \\ 0 & -5 & 8-a & 0 \\ 0 & 0 & -12-6a & 0 \end{array}\right]$$

· para ter uma única solução $\rightarrow$ SPD, logo $-12-6a \neq 0 \Leftrightarrow a \neq -2$
(pois é homogéneo)

④ Determine qual a relação entre a, b e c tal que o sistema ... admite soluções não nulas

$$\begin{cases} 2x+3y-2z=0 \\ x-2y+4z=0 \\ ax+by+cz=0 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} 2 & 3 & -2 & 0 \\ 1 & -2 & 4 & 0 \\ a & b & c & 0 \end{array}\right] \underset{\Leftrightarrow}{L_3 \leftrightarrow L_1} \left[\begin{array}{ccc|c} a & b & c & 0 \\ 1 & -2 & 4 & 0 \\ 2 & 3 & -2 & 0 \end{array}\right]$$

$$\begin{array}{c} aL_3-2L_1 \\ \Leftrightarrow \\ aL_2-L_1 \end{array} \left[\begin{array}{ccc|c} a & b & c & 0 \\ 0 & -2a-b & 4a-c & 0 \\ 0 & 3a-2b & -2a-2c & 0 \end{array}\right] \Leftrightarrow \left[\begin{array}{ccc|c} a & c & b & 0 \\ 0 & 4a-c & -2a-b & 0 \\ 0 & 2a-2c & 3a-2b & 0 \end{array}\right] \underset{\Leftrightarrow}{2L_3-L_2} \left[\begin{array}{ccc} a & c & b \\ 0 & & \\ 0 & -4c & \end{array}\right]$$

Sistemas de equações

exercícios

1.1

1. a)

$$\begin{cases} 5x+3y=-7 \\ 4x+5y=-3 \end{cases} \Leftrightarrow \left[\begin{array}{cc|c} 5 & 3 & -7 \\ 4 & 5 & -3 \end{array}\right] \Leftrightarrow \left[\begin{array}{cc|c} 4 & 5 & -3 \\ 5 & 3 & -7 \end{array}\right] \overset{4L_2-5L_1}{\Leftrightarrow} \left[\begin{array}{cc|c} 4 & 5 & -3 \\ 0 & -13 & -13 \end{array}\right] \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 4x+5y=-3 \\ +13y=+13 \end{cases} \Rightarrow \begin{cases} x=-2 \\ y=1 \end{cases} \Leftrightarrow (x,y)=(-2,1) \quad \underline{\text{SPD}}$$

b)

$$\begin{cases} x+2y-z=3 \\ x+3y+z=5 \\ 3x+8y+4z=17 \end{cases} \begin{array}{c} L_3-3L_1 \\ \Leftrightarrow \\ L_2-L_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 2 & 7 & 8 \end{array}\right] \Leftrightarrow$$

$$\overset{L_3-2L_2}{\Leftrightarrow} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 3 & 4 \end{array}\right] \Leftrightarrow \begin{cases} x+2y-z=3 \\ y+2z=2 \\ z=\frac{4}{3} \end{cases} \Leftrightarrow \begin{cases} x=\frac{17}{3} \\ y=-\frac{2}{3} \end{cases} \quad \underline{\text{SPD}}$$

c)

$$\begin{cases} 2x+y-2z=-2 \\ 3x-2y+z=2 \\ -6x-y+4z=4 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} 2 & 1 & -2 & -2 \\ 3 & -2 & 1 & 2 \\ -6 & -1 & 4 & 4 \end{array}\right] \begin{array}{c} L_3+2L_2 \\ \Leftrightarrow \\ 2L_2-3L_1 \end{array} \left[\begin{array}{ccc|c} 2 & 1 & -2 & -2 \\ 0 & -7 & 8 & 10 \\ 0 & -5 & 6 & 8 \end{array}\right] \Leftrightarrow$$

$$\overset{7L_3-5L_2}{\Leftrightarrow} \left[\begin{array}{ccc|c} 2 & 1 & -2 & -2 \\ 0 & -7 & 8 & 10 \\ 0 & 0 & 2 & 6 \end{array}\right] \Leftrightarrow \begin{cases} 2x+y-2y=-2 \\ -7y+8z=10 \\ z=3 \end{cases} \Leftrightarrow \begin{cases} x=1 \\ y=2 \\ z=3 \end{cases} \Leftrightarrow (x,y,z)=(1,2,3) \quad \underline{\text{SPD}}$$

(utilizando método da adição ordenada)

d)

$$\begin{cases} x-y+z-w=2 \\ x-y+z+w=0 \\ 4x-4y+4z=4 \\ -2x+2y-2z+w=-3 \end{cases} \begin{array}{c} L_4+2L_1 \\ \Leftrightarrow \\ L_3-4L_1 \\ L_2-L_1 \end{array} \begin{cases} x-y+z-w=2 \\ 2w=-2 \\ +4w=-4 \\ +w=-1 \end{cases} \Leftrightarrow \begin{cases} x-y+z=1 \\ w=-1 \\ w=-1 \\ w=-1 \end{cases} \Leftrightarrow (x,y,w,z)=(x,y,1,1-x+y) \quad \underline{\text{SPI}}$$

2. a)

$$\begin{cases} x+2y+z=2 \\ 2x+3y+4z=1 \\ -x+y+4z=b \end{cases} \begin{array}{c} L_3+L_1 \\ \Leftrightarrow \\ L_2-2L_1 \end{array} \begin{cases} x+2y+z=2 \\ -y+2z=-3 \\ 3y+(az+z)=(b+2) \end{cases} \Leftrightarrow \left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 0 & -1 & 2 & -3 \\ 0 & 3 & a+1 & b+2 \end{array}\right] \Leftrightarrow$$

$$\overset{L_3+3L_1}{\Leftrightarrow} \left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 0 & -1 & 2 & -3 \\ 0 & 0 & a+7 & b-7 \end{array}\right]$$

logo, SPD se $a+7 \neq 0 \Leftrightarrow a \neq -7$ sol. única

SPI se $a+7=0 \wedge b-7=0 \Leftrightarrow a=-7 \wedge b=7$ $(0=0)$

SI se $a+7=0 \wedge b-7 \neq 0 \Leftrightarrow a=-7 \wedge b \neq 7$ $(0=K)$

b) $\begin{cases} x+y+az=1 \\ x+3y+(a+2)z=b+1 \\ ax+(a-4)y=a-b-3 \end{cases} \Leftrightarrow \begin{bmatrix} 1 & 1 & a & 1 \\ 1 & 3 & a+2 & b+1 \\ a & a-4 & 0 & a-b-3 \end{bmatrix} \underset{L_2-L_1}{\overset{L_3-aL_1}{\Leftrightarrow}} \begin{bmatrix} 1 & 1 & a & 1 \\ 0 & 2 & 2 & b \\ 0 & -4 & -a^2 & -b-3 \end{bmatrix}$

$\overset{L_3+2L_2}{\Leftrightarrow} \begin{bmatrix} 1 & 1 & a & 1 \\ 0 & 2 & 2 & b \\ 0 & 0 & -a^2+4 & +b-3 \end{bmatrix}$

logo, SPD se $+a^2+4 \neq 0 \Leftrightarrow a = \sqrt{+4} \Leftrightarrow a \neq 2 \vee a \neq -2$

SPI se $(a=2 \vee a=-2) \wedge (b=3 \vee \underbrace{b=0}_{\text{ñ preciso}})$

SI se $(a=2 \vee a=-2) \wedge b \neq 3$

c) $\begin{cases} x-y+2z=1 \\ x+y+3z=3 \\ -3x+5y+(a^2-6)z=ab+b \end{cases} \Leftrightarrow \begin{bmatrix} 1 & -1 & 2 & 1 \\ 1 & 1 & 3 & 3 \\ -3 & 5 & a^2-6 & ab+b \end{bmatrix} \underset{L_2-L_1}{\overset{L_3+3L_1}{\Leftrightarrow}} \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 2 & a^2 & ab+b+3 \end{bmatrix}$

$\overset{L_3-L_2}{\Leftrightarrow} \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & a^2-1 & ab+b+1 \end{bmatrix}$

logo, SPD se $a^2 \neq 1 \Leftrightarrow a \neq 1 \vee a \neq -1$

• se $a=1$ $\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2b+1 \end{bmatrix}$

SPI se $a=1 \wedge 2b+1=0 \Leftrightarrow a=1 \wedge b=-\frac{1}{2}$

SI se $a=1 \wedge b \neq -\frac{1}{2}$

• se $a=-1$ $\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

SI se $a=-1$ (pois $0=k$, i.e, $0=1$)

d) $\begin{cases} x-y+z=2 \\ x-(a+1)y-az=1 \\ ax+(a+3)z=2a-b-2 \end{cases} \Leftrightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 1 & -a-1 & -a & 1 \\ a & 0 & a+3 & 2a-b-2 \end{bmatrix} \underset{L_2-L_1}{\overset{L_3-aL_1}{\Leftrightarrow}} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & -a & -a-1 & -1 \\ 0 & a & 3 & -b-2 \end{bmatrix}$

$\overset{L_3+L_2}{\Leftrightarrow} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & -a & -a-1 & -1 \\ 0 & 0 & 2-a & -b-3 \end{bmatrix}$

SPD se $-a \neq 0 \wedge 2-a \neq 0 \Leftrightarrow a \neq 0 \wedge a \neq 2$

• se $a=0$ $\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 2 & -b-3 \end{bmatrix} \overset{L_3+2L_2}{\Leftrightarrow} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -b-5 \end{bmatrix}$

SPI se $a=0 \wedge -b-5=0 \Leftrightarrow a=0 \wedge b=-5$

SI se $a=0 \wedge b \neq -5$

• se $a=2$ $\left[\begin{array}{cccc} 1 & -1 & 1 & 2 \\ 0 & -2 & -3 & -1 \\ 0 & 0 & 0 & -b-3 \end{array}\right]$ SPI se $a=2 \wedge b=-3$

SI se $a=2 \wedge b \neq 3$

2) $\begin{cases} x+y-az=0 \\ x+ay=1 \\ ax+(2-a)y-(2a+4)z=b-2 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 1 & a & 0 & 1 \\ a & 2-a & -2a-4 & b-2 \end{array}\right] \underset{L_2-L_1}{\overset{L_3-aL_1}{\Leftrightarrow}} \left[\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & a-1 & a & 1 \\ 0 & 2-2a & a^2-2a-4 & b-2 \end{array}\right]$

$\overset{L_3+2L_2}{\Leftrightarrow} \left[\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & a-1 & a & 1 \\ 0 & 0 & a^2-4 & b-2 \end{array}\right]$ logo, SPD se $a-1\neq 0 \wedge a^2-4 \neq 0 \Leftrightarrow a\neq 1 \wedge a\neq 2 \wedge a \neq -2$

• se $a=1$ $\left[\begin{array}{cccc} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -3 & b \end{array}\right] \overset{L_3+3L_2}{\Leftrightarrow} \left[\begin{array}{cccc} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & b+3 \end{array}\right]$ SPI se $a=1 \wedge b=-3$

SI se $a=1 \wedge b\neq -3$

• se $a=2$ $\left[\begin{array}{cccc} 1 & 1 & -2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & b-2 \end{array}\right]$ SPI se $a=2 \wedge b$ 0

SI se $a=2 \wedge b\neq 0$

• se $a=-2$. $\left[\begin{array}{cccc} 1 & 1 & 2 & 0 \\ 0 & -3 & -2 & 1 \\ 0 & 0 & 0 & b \end{array}\right]$ SPI se $a=-2 \wedge b=0$

SI se $a=-2 \wedge b=0$

3) $\begin{cases} x-y+az=1 \\ -x+(a+1)y=b+1 \\ ax+2a^2z=a+2b-1 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} 1 & -1 & a & 1 \\ -1 & a+1 & 0 & b+1 \\ a & 0 & 2a^2 & a+2b-1 \end{array}\right] \underset{L_2+L_1}{\overset{L_3-aL_1}{\Leftrightarrow}} \left[\begin{array}{ccc|c} 1 & -1 & a & 1 \\ 0 & a & a & b+2 \\ 0 & +a & a^2 & 2b-1 \end{array}\right]$

$\overset{L_3-L_2}{\Leftrightarrow} \left[\begin{array}{ccc|c} 1 & -1 & a & 1 \\ 0 & a & a & b+2 \\ 0 & 0 & a^2-a & b-3 \end{array}\right]$ logo, SPD se $a\neq 0 \wedge a^2-a\neq 0 \Leftrightarrow a\neq 0 \wedge a(a-1)\neq 0$

$\Leftrightarrow a\neq 0 \wedge a\neq 1$

? • se $a=0$ $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & b+2 \\ 0 & 0 & 0 & b-3 \end{array}\right]$ SPI se $a=0 \wedge b=3$

SI se $a=0 \wedge b\neq 3$

Agenda: 24/11 in na aula
3ª 12h30 - 14h ; S12

- se $a = 1$ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & b+2 \\ 0 & 0 & 0 & b-3 \end{array}\right]$ SPI se $a=1 \wedge b=3$

SI se $a=1 \wedge b \neq 3$

g) $\begin{cases} x+y-az=0 \\ x+(a^2-3)y+2z=b \\ ax+(2a^2+a-8)y+4z=3b+1 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 1 & a^2-3 & 2 & b \\ a & 2a^2+a-8 & 4 & 3b+1 \end{array}\right] \Leftrightarrow$

$\underset{L_2-L_1}{\overset{L_3-aL_1}{\Leftrightarrow}} \left[\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & a^2-4 & 2+a & b \\ 0 & 2a^2-8 & 4+a^2 & 3b+1 \end{array}\right] \overset{L_3-2L_2}{\Leftrightarrow} \left[\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & a^2-4 & a+2 & b \\ 0 & 0 & -a^2-2a & b+1 \end{array}\right]$

- SPD se $a^2-4 \neq 0 \wedge a^2-2a \neq 0 \Leftrightarrow (a \neq 2 \wedge a \neq -2) \wedge a \neq 0$

- se $a=2$ $\left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 0 & 4 & b \\ 0 & 0 & -8 & b+1 \end{array}\right]$ SPI se $a=2 \wedge -8=b+1 \Leftrightarrow a=2 \wedge b=-9$

SI se $a=2 \wedge b \neq -9$

- se $a=-2$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & b \\ 0 & 0 & 8 & b+1 \end{array}\right]$ SPI se $a=-2 \wedge b \neq 0$

SI se $a=2 \wedge b \neq 0$

$\overset{L_3 \leftrightarrow L_2}{\Leftrightarrow} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 0 & 8 & b+1 \\ 0 & 0 & 0 & b \end{array}\right]$

- se $a=0$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -4 & 2 & b \\ 0 & 0 & 0 & b+1 \end{array}\right]$ SPI se $a=0 \wedge b=-1$

SI se $a=0 \wedge b \neq -1$

h) $\begin{cases} x-y+az+2w=1 \\ -x-y+w=\beta \\ 2y+z-3w=0 \\ x+y-w=1 \end{cases} \Leftrightarrow \left[\begin{array}{cccc|c} 1 & -1 & a & 2 & 1 \\ -1 & -1 & 0 & 1 & \beta \\ 0 & 2 & 1 & -3 & 0 \\ 1 & 1 & 0 & -1 & 1 \end{array}\right] \underset{L_2+L_1}{\overset{L_4-L_1}{\Leftrightarrow}} \left[\begin{array}{cccc|c} 1 & -1 & a & 2 & 1 \\ 0 & -2 & a & 3 & \beta+1 \\ 0 & 2 & 1 & -3 & 0 \\ 0 & 2 & -a & -3 & 0 \end{array}\right]$

$\underset{L_3+L_2}{\overset{L_4-L_3}{\Leftrightarrow}} \left[\begin{array}{cccc|c} 1 & -1 & a & 2 & 1 \\ 0 & -2 & a & 3 & \beta+1 \\ 0 & 0 & 1+a & 0 & \beta+1 \\ 0 & 0 & -a-1 & 0 & 0 \end{array}\right] \overset{L_4+L_3}{\Leftrightarrow} \left[\begin{array}{cccc|c} 1 & -1 & a & 2 & 1 \\ 0 & -2 & a & 3 & \beta+1 \\ 0 & 0 & 1+a & 0 & \beta+1 \\ 0 & 0 & 0 & 0 & \beta+1 \end{array}\right]$ - SI se $\beta \neq -1$

$-a-1$

by Staples

. SPI se $\beta = -1$ $\left[\begin{array}{cccc|c} 1 & -1 & a & 2 & 1 \\ 0 & -2 & a & 3 & 1 \\ 0 & 0 & 1+a & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$. se $\beta = -1$ e $a \neq -1 \rightarrow$ SPI

3. a) $\begin{cases} x + y + 2z + t = 1 \\ -x + y - z + (a-1)t = 1 \\ 2x + 4y + (a+4)z + 3at = 4 \end{cases} \Leftrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ -1 & 1 & -1 & a-1 & 1 \\ 2 & 4 & a+4 & 3a & 4 \end{array}\right] \Leftrightarrow$

$\underset{L_2+L_1}{\overset{L_3-2L_1}{\Leftrightarrow}} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ 0 & 2 & 1 & a & 2 \\ 0 & 2 & a & 3a-2 & 2 \end{array}\right] \overset{L_3-L_2}{\Leftrightarrow} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ 0 & 2 & 1 & a & 2 \\ 0 & 0 & a-1 & 2a-2 & 0 \end{array}\right]$. SPI se $a-1=0 \Leftrightarrow a=1$

. se $a=1$ $\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ 0 & 2 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$. se $a=1$ SPI

b) $\begin{cases} x + 2y - z - t = 1 \\ x + 3y + z = 2 \\ 2x + 2y + (a-5)z - (a+2)t = 2 \end{cases} \Leftrightarrow \left[\begin{array}{cccc|c} 1 & 2 & -1 & -1 & 1 \\ 1 & 3 & 1 & 0 & 2 \\ 2 & 2 & a-5 & -a-2 & 2 \end{array}\right] \Leftrightarrow$

$\underset{L_2-L_1}{\overset{L_3-2L_1}{\Leftrightarrow}} \left[\begin{array}{cccc|c} 1 & 2 & -1 & -1 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & -2 & a-3 & -a & -2 \end{array}\right] \Leftrightarrow [$

$a-5+2$ $\quad -a-2+2$

capítulo 3

3.1.1. a) $\begin{bmatrix} 1 & 3 \\ 5 & 8 \end{bmatrix}$ $\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 5 & 8 & 0 & 1 \end{array}\right] \overset{L_2-5L_1}{\Leftrightarrow} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -7 & -5 & 1 \end{array}\right] \overset{7L_1+3L_2}{\Leftrightarrow} \left[\begin{array}{cc|cc} 7 & 0 & -8 & 3 \\ 0 & -7 & -5 & 1 \end{array}\right]$

b) $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ $\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 6 & 0 & 1 \end{array}\right] \overset{L_2-3L_1}{\Leftrightarrow} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ \boxed{0 \quad 0} & & 3 & 1 \end{array}\right]$ impossível

c) $\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \underset{2L_2-L_1}{\overset{2L_3-L_1}{\Leftrightarrow}} \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3 & 1 & -1 & 2 & 0 \\ 0 & 1 & 3 & -1 & 0 & 2 \end{array}\right] \overset{3L_3-L_2}{\Leftrightarrow} \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3 & 1 & -1 & 2 & 0 \\ 0 & 0 & 8 & -2 & -2 & 6 \end{array}\right]$

$\underset{8L_1-L_3}{\overset{8L_2-L_3}{\Leftrightarrow}} \left[\begin{array}{ccc|ccc} 16 & 8 & 0 & 2 & -18 & 6 \\ 0 & 24 & 0 & -6 & 18 & -6 \\ 0 & 0 & 8 & -2 & -2 & 6 \end{array}\right] \overset{3L_1-L_2}{\Leftrightarrow} \left[\begin{array}{ccc|ccc} 48 & 0 & 0 & 0 & 36 & 12 \\ 0 & 24 & 0 & -6 & 18 & -6 \\ 0 & 0 & 8 & -2 & -2 & 6 \end{array}\right]$

① Discute o sistema em função dos respectivos parâmetros reais ($\alpha, \beta \in \mathbb{R}$)

a) $\begin{cases} x + y + \alpha z = 2 \\ 3x + 4y + 2z = \beta \\ 2x + 3y - z = 1 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & \alpha & 2 \\ 3 & 4 & 2 & \beta \\ 2 & 3 & -1 & 1 \end{array}\right] \underset{L_3-2L_1}{\overset{L_2-3L_1}{\Leftrightarrow}} \left[\begin{array}{ccc|c} 1 & 1 & \alpha & 2 \\ 0 & 1 & 2-3\alpha & \beta-6 \\ 0 & 1 & -1-2\alpha & -3 \end{array}\right]$

$\underset{L_3-L_2}{\Leftrightarrow} \left[\begin{array}{ccc|c} 1 & 1 & \alpha & 2 \\ 0 & 1 & 2-3\alpha & \beta-6 \\ 0 & 0 & -3+\alpha & 3-\beta \end{array}\right]$. SPD se $1 \neq 0 \wedge 1 \neq 0 \wedge -3+\alpha \neq 0 \Leftrightarrow \alpha \neq 3$

$\boxed{\text{se } \alpha = 3}$ $\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & -7 & \beta-6 \\ 0 & 0 & 0 & 3-\beta \end{array}\right]$ $\to \beta = 3$ SPI grau ind. 1 $\to (x, 2x, 1)$ ou grau liberdade 2 $(x+y, y, 2)$

$\to \beta \neq 3$ SI

conclusão: SPD: $\alpha \neq 3$ SPI: $\beta = 3 \wedge \alpha = 3$ SI: $\beta \neq 3 \wedge \alpha = 3$

b) $\begin{cases} x + 2y + z = 2 \\ x + 2\alpha y + 3z = 5 \\ 3x + (\alpha+5)y + \alpha^2 z = \beta + 8 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 1 & 2\alpha & 3 & 5 \\ 3 & \alpha+5 & \alpha^2 & \beta+8 \end{array}\right] \underset{L_3-3L_1}{\overset{L_2-L_1}{\Leftrightarrow}} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2\alpha-2 & 2 & 3 \\ 0 & \alpha+1 & \alpha^2-3 & \beta+2 \end{array}\right]$

$\overset{2L_3-L_2}{\Leftrightarrow} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2\alpha-2 & 2 & 3 \\ 0 & 0 & 2\alpha^2-8 & 2\beta+1 \end{array}\right]$ SPD: $2\alpha - 2 \neq 0 \wedge 2\alpha^2 - 8 \neq 0 \Leftrightarrow \alpha \neq 1 \wedge \alpha \neq \pm 2$

$\boxed{\text{se } \alpha = 1}$ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & -6 & 2\beta+1 \end{array}\right] \overset{L_3+3L_2}{\Leftrightarrow} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 2\beta+10 \end{array}\right]$ $\to$ SPI: $2\beta + 10 = 0 \Leftrightarrow \beta = -5$

$\to$ SI: $2\beta + 10 \neq 0 \Leftrightarrow \beta \neq -5$

continuar a condensar

$\boxed{\text{se } \alpha = 2}$ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & 2 & 3 \\ 0 & 0 & 0 & 2\beta+1 \end{array}\right]$ $\to$ SPI: $2\beta + 1 = 0 \Leftrightarrow \beta = \frac{-1}{2}$

$\to$ SI: $2\beta + 1 \neq 0 \Leftrightarrow \beta \neq \frac{-1}{2}$

$\boxed{\text{se } \alpha = -2}$ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -6 & 2 & 3 \\ 0 & 0 & 0 & 2\beta+1 \end{array}\right]$ $\to$ SPI: $\beta = -\frac{1}{2}$

$\to$ SI: $\beta \neq -\frac{1}{2}$

conclusão: SPD: $\alpha \neq 1 \wedge \alpha \neq 2 \wedge \alpha \neq -2$ SI: $(\alpha = 1 \wedge \beta \neq -5) \vee (\alpha = 2 \wedge \beta \neq \frac{-1}{2}) \vee (\alpha = -2 \wedge \beta \neq -\frac{1}{2})$

SPI: $(\alpha = 1 \wedge \beta = -5) \vee (\alpha = 2 \wedge \beta = -\frac{1}{2}) \vee (\alpha = -2 \wedge \beta = -\frac{1}{2})$

c) $\begin{cases} x + y + \alpha z = \alpha^2 \\ x + \alpha y + z = \alpha \\ \alpha x + y + z = 1 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & \alpha & \alpha^2 \\ 1 & \alpha & 1 & \alpha \\ \alpha & 1 & 1 & 1 \end{array}\right] \underset{L_2 - L_1}{\overset{L_3 - \alpha L_1}{\Leftrightarrow}} \left[\begin{array}{ccc|c} 1 & 1 & \alpha & \alpha^2 \\ 0 & \alpha - 1 & 1-\alpha & \alpha - \alpha^2 \\ 0 & 1-\alpha & 1-\alpha^2 & 1-\alpha^3 \end{array}\right]$

$\overset{L_3 + L_2}{\Leftrightarrow} \left[\begin{array}{ccc|c} 1 & 1 & \alpha & \alpha^2 \\ 0 & \alpha - 1 & 1-\alpha & \alpha - \alpha^2 \\ 0 & 0 & 2-\alpha^2-\alpha & 1+\alpha-\alpha^2-\alpha^3 \end{array}\right]$

. SPD se $\alpha - 1 \neq 0 \wedge -\alpha^2 - \alpha + 2 \neq 0$

$\Leftrightarrow \alpha \neq 1 \wedge \alpha \neq \frac{1 \pm \sqrt{1 - 4 \times (-1) \times 2}}{-2}$

$\Leftrightarrow \alpha \neq 1 \wedge \alpha \neq \frac{1 \pm \sqrt{9}}{-2}$

$\Leftrightarrow \alpha \neq 1 \wedge (\alpha \neq -2 \vee \alpha \neq +1)$

. se $\alpha = 1$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ SPI g.i. 2

. se $\alpha = -2$ $\left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & -3 & 3 & 2 \\ 0 & 0 & 0 & 3 \end{array}\right]$. SI se $\alpha = -2$

d) $\begin{cases} \alpha x + y + (\alpha - 1) z = \beta \\ x + \alpha y + z = 1 \\ \alpha x + y - z = 0 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} \alpha & 1 & \alpha - 1 & \beta \\ 1 & \alpha & 1 & 1 \\ \alpha & 1 & -1 & 0 \end{array}\right] \overset{L_3 - L_1}{\Leftrightarrow} \left[\begin{array}{ccc|c} \alpha & 1 & \alpha - 1 & \beta \\ 1 & \alpha & 1 & 1 \\ 0 & 0 & -\alpha & 0 \end{array}\right]$

$\overset{L_2 \leftrightarrow L_1}{\Leftrightarrow} \left[\begin{array}{ccc|c} 1 & \alpha & 1 & 1 \\ \alpha & 1 & \alpha - 1 & \beta \\ 0 & 0 & -\alpha & 0 \end{array}\right] \overset{L_2 - \alpha L_1}{\Leftrightarrow} \left[\begin{array}{ccc|c} 1 & \alpha & 1 & 1 \\ 0 & 1-\alpha^2 & -1 & \beta - \alpha \\ 0 & 0 & -\alpha & 0 \end{array}\right]$

SPD se $1 - \alpha^2 \neq 0 \wedge -\alpha \neq 0$

$\Leftrightarrow \alpha \neq \pm 1 \wedge \alpha \neq 0$

$\alpha \neq -2$

SPI $\begin{cases} \alpha = 1 \wedge \beta = \frac{3}{2} \\ \alpha = -2 \wedge \beta = 0 \end{cases}$

SI $\alpha = -1 \wedge$

$\alpha = 1 \wedge \beta \neq \frac{3}{2}$

$\alpha = 2 \wedge \beta \neq 0$

. se $\alpha = 0$ $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & \beta - 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ SPI

. se $\alpha = 1$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -1 & \beta - 2 \\ 0 & 0 & -1 & 0 \end{array}\right] \Leftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -1 & \\ & & & \end{array}\right]$

. se $\alpha = -1$ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 0 & -1 & \beta - 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$

2. Discute os sistemas para os parâmetros reais $(a, b \in \mathbb{R})$

a) $\begin{cases} x + ay + az = 0 \\ x + y + az + (a-1)w = b \\ ax + (a-1)y + w = 1 \end{cases} \Leftrightarrow \left[\begin{array}{cccc|c} 1 & a & a & 0 & 0 \\ 1 & 1 & a & a-1 & b \\ a & a-1 & 0 & 1 & 1 \end{array}\right] \underset{\Leftrightarrow}{C_1 \leftrightarrow C_3} \left[\begin{array}{cccc|c} a & a & 1 & 0 & 0 \\ a & 1 & 1 & a-1 & b \\ 0 & a-1 & a & 1 & 1 \end{array}\right] \Leftrightarrow$

$\underset{\Leftrightarrow}{L_2 - L_1} \left[\begin{array}{cccc|c} a & a & 1 & 0 & 0 \\ 0 & 1-a & 0 & a-1 & b \\ 0 & a-1 & a & 1 & 1 \end{array}\right] \underset{\Leftrightarrow}{L_3 + L_2} \left[\begin{array}{cccc|c} a & a & 1 & 0 & 0 \\ 0 & 1-a & 0 & a-1 & b \\ 0 & 0 & a & a & b+1 \end{array}\right]$

• SPI: $a \neq 0 \land 1-a \neq 0 \land a \neq 0$

$\Leftrightarrow a \neq 0 \land a \neq 1$

g.i. = 1

• se $a = 0$ $\left[\begin{array}{cccc|c} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & b \\ 0 & 0 & 0 & 0 & \boxed{b+1} \end{array}\right]$
$b \neq -1 \rightarrow$ SI
$b = -1 \rightarrow$ SPI g.i. = 2

• se $a = 1$ $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \boxed{b} \\ 0 & 0 & 1 & 1 & b+1 \end{array}\right]$
$b \neq 0 \rightarrow$ SI
$b = 0 \rightarrow$ SPI g.i. = 2

conclusão SPI: $(a \neq 0 \land a \neq 1) \lor (a = 0 \land b = -1) \lor (a = 1 \land b = 0)$

SI: $(a = 0 \land b \neq -1) \lor (a = 1 \land b \neq 0)$

b) $\begin{cases} x - y + 2z = 1 \\ x + az = 1 \\ x + y + 2z = b \\ 2x - y + (a+2)z = 2 \end{cases} \quad \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 1 & 0 & a & 1 \\ 1 & 1 & 2 & b \\ 2 & -1 & a+2 & 2 \end{array}\right] \underset{\Leftrightarrow}{C_1 \leftrightarrow C_2} \left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 1 & a & 1 \\ 1 & 1 & 2 & b \\ -1 & 2 & a+2 & 2 \end{array}\right] \Leftrightarrow$

$\underset{L_4 - L_1}{\overset{L_3 + L_1}{\Leftrightarrow}} \left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 1 & a & 1 \\ 0 & 2 & 4 & b+1 \\ 0 & 1 & a & 1 \end{array}\right] \underset{L_4 - L_2}{\overset{L_3 - 2L_2}{\Leftrightarrow}} \left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 1 & a & 1 \\ 0 & 0 & 4-2a & b-1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

SPD: $a \neq 2$

• se $a = 2$ $\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & b-1 \end{array}\right]$
$b \neq 1$ SI
$b = 1$ SPI g.i. = 1

conclusão: $a \neq 2 \rightarrow$ SPD

$a = 2 \land b = 1 \rightarrow$ SPI

$a = 2 \land b \neq 1 \rightarrow$ SI

se fosse $\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 1 & a & 1 \\ 0 & 0 & 4-2a & b-1 \\ 0 & 0 & 0 & b \end{array}\right]$ $\begin{cases} b \neq 0 & SI \\ b = 0 & ? \end{cases}$ se $b=0$ $\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 1 & a & 1 \\ 0 & 0 & 4-2a & -1 \end{array}\right]$ $a \neq 2$

$SI: b \neq 0 \vee (b=0 \wedge a=2)$

$SPD: b=0 \wedge a \neq 2$

c) $\begin{cases} x+y+2z=b \\ x-y+2z=1 \\ x+az=1 \\ 2x-y+(a+2)z=b+4 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & b \\ 1 & -1 & 2 & 1 \\ 1 & 0 & a & 1 \\ 2 & -1 & a+2 & b+4 \end{array}\right]$ $\underset{C_2 \leftrightarrow C_1}{}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & b \\ -1 & 1 & 2 & 1 \\ 0 & 1 & a & 1 \\ -1 & 2 & a+2 & b+4 \end{array}\right]$

$\underset{L_2+L_1}{\overset{L_4+L_1}{\Leftrightarrow}}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & b \\ 0 & 2 & 4 & 1+b \\ 0 & 1 & a & 1 \\ 0 & 3 & a+4 & 2b+4 \end{array}\right]$ $\underset{2L_3+L_2}{\overset{-2L_4+3L_2}{\Leftrightarrow}}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & b \\ 0 & 2 & 4 & 1+b \\ 0 & 0 & -2a+4 & -1+b \\ 0 & 0 & -2a+4 & -b+5 \end{array}\right]$ $\underset{L_4-L_3}{\Leftrightarrow}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & b \\ 0 & 2 & 4 & 1+b \\ 0 & 0 & -2a+4 & -1+b \\ 0 & 0 & 0 & -2b-4 \end{array}\right]$

. se $-2b-4 \neq 0 \Leftrightarrow b \neq -2 \rightarrow SI$

se $b=2$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & 2 & 4 & -1 \\ 0 & 0 & -2a+4 & -3 \end{array}\right]$ $SPD: b=-2 \wedge a \neq 2$

$SI: b=-2 \wedge a=2$

d) $\begin{cases} x+y=1 \\ ay+z=1 \\ -x-y=b \\ x+(a+1)y+az=2 \end{cases}$

$SPD: b=-1 \wedge a \neq 0 \wedge a \neq 1$

$SPI: b=-1 \wedge a=1$

$SI: b \neq -1 \vee (b=-1 \wedge a=0)$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & a & 1 & 1 \\ -1 & -1 & 0 & b \\ 1 & a+1 & a & 2 \end{array}\right]$ $\underset{\Leftrightarrow}{L_4 \leftrightarrow L_2}$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & a+1 & a & 2 \\ -1 & -1 & 0 & b \\ 0 & a & 1 & 1 \end{array}\right]$ $\Leftrightarrow$

$\underset{L_2-L_1}{\overset{L_3+L_1}{\Leftrightarrow}}$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & a & a & 1 \\ 0 & 0 & 0 & b+1 \\ 0 & a & 1 & 1 \end{array}\right]$ $\underset{\Leftrightarrow}{L_4 \leftrightarrow L_3}$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & a & a & 1 \\ 0 & a & 1 & 1 \\ 0 & 0 & 0 & b+1 \end{array}\right]$ $\underset{\Leftrightarrow}{L_3-L_2}$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & a & a & 1 \\ 0 & 0 & 1-a & 0 \\ 0 & 0 & 0 & b+1 \end{array}\right]$ $\begin{cases} b \neq -1 & SI \\ b = 1 & ? \end{cases}$

. se $b=-1$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & a & a & 1 \\ 0 & 0 & 1-a & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

. $a \neq 0 \wedge a \neq 1 \rightarrow SPD$

. $a=0 \rightarrow SI$

. $a=1 \rightarrow$

conclusão

. SPD se $b=-1 \wedge a \neq 0 \wedge a \neq 1$

. SI se $b \neq 1 \vee b=-1 \wedge a=0$

. SPI se $b=-1 \wedge a=1$

1. Construa as matrizes $A = (a_{ij})_{2\times 3}$ onde

a) $a_{ij} = i + j$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

b) $a_{ij} = (-1)^{i+j}$

$$A = \begin{bmatrix} (-1)^2 & (-1)^3 & (-1)^4 \\ (-1)^3 & (-1)^4 & (-1)^5 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

2. Calcule $3A + B - 2C$ sendo $A = \begin{bmatrix} 2 & 3 \\ 4 & 2 \\ 5 & 6 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 2 \\ 2 & 5 \\ 3 & 3 \end{bmatrix}$ e $C = \begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 4 & 6 \end{bmatrix}$

$$3A + B - 2C = \begin{bmatrix} 6 & 9 \\ 12 & 6 \\ 15 & 18 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 5 \\ 3 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 6 & 10 \\ 8 & 12 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 8 & 1 \\ 10 & 9 \end{bmatrix}$$

3. Determine a matriz B tal que $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + 2B = \begin{bmatrix} 0 & 1 \\ 3 & 5 \end{bmatrix}$ ou $2B = \begin{bmatrix} 0 & 1 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$$\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + 2\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & 5 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 3+2a & 1+2b \\ 2+2c & 1+2d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & 5 \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} 3+2a = 0 \\ 1+2b = 1 \\ 2+2c = 3 \\ 1+2d = 5 \end{cases} \Leftrightarrow \begin{cases} a = -\frac{3}{2} \\ b = 0 \\ c = \frac{1}{2} \\ d = 2 \end{cases} \qquad B = \begin{bmatrix} -\frac{3}{2} & 0 \\ \frac{1}{2} & 2 \end{bmatrix}$$

4. Resolva a equação matricial

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} 3x = x+4 \\ 3y = 6+x+y \\ 3z = -1+z+w \\ 3w = 2w+3 \end{cases} \Leftrightarrow \begin{cases} x = 2 \\ 3y = 6+2+y \\ 2z = 2 \\ w = 3 \end{cases} \Leftrightarrow \begin{cases} x = 2 \\ y = 4 \\ z = 1 \\ w = 3 \end{cases} \Leftrightarrow (x, y, z, w) = (2, 4, 1, 3)$$

5. Calcule AB e BA sendo $A = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$ e $B = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2-3 & -1 \\ 10+6 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 16 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2+0 & -2 \\ 3+5 & -3+2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 8 & -1 \end{bmatrix}$$

6. Determine a matriz M tal que AM = B sendo $A = \begin{bmatrix} 2 & 0 \\ 3 & 1 \\ 4 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 8 \\ 13 \\ 15 \end{bmatrix}$

$$\underset{3\times 2}{\begin{bmatrix} 2 & 0 \\ 3 & 1 \\ 4 & -1 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} a \\ b \end{bmatrix}} = \underset{3\times 1}{\begin{bmatrix} 8 \\ 13 \\ 15 \end{bmatrix}} \Leftrightarrow \begin{bmatrix} 2a \\ 3a+b \\ 4a-b \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \\ 15 \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} a = 4 \\ 3a+b = 13 \\ 4a-b = 15 \end{cases} \Leftrightarrow \begin{cases} a = 4 \\ b = 1 \end{cases} \qquad M = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

7. Calcule A^2 e A^3 sendo $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1-4 & 2+2 \\ -2-2 & -4+1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ -4 & -3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -3 & 4 \\ -4 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3-8 & -6+4 \\ -4+6 & -8-3 \end{bmatrix} = \begin{bmatrix} -11 & -2 \\ 2 & -11 \end{bmatrix}$$

8. Sendo a matriz $A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$, determine as matrizes B quadradas não nulas que verifiquem

a) AB = 0

$$\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} c & d \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{cases} c = 0 \\ d = 0 \end{cases}$$

$B = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$, $a, b \in \mathbb{R}$ exceepto a e b igual a zero simultaneamente

9. Determine as matrizes que são permutáveis com a matriz $A = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$

$AB = BA$

$$\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} a+2c & b+2d \\ -a-c & -b-d \end{bmatrix} = \begin{bmatrix} a-b & 2a-b \\ c-d & 2c-d \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} a+2c = a-b \\ b+2d = 2a-b \\ -a-c = c-d \\ -b-d = 2c-d \end{cases} \Leftrightarrow \begin{cases} b = -2c \\ b = a-d \\ -2c = a-d \\ b = 2c \end{cases} \Leftrightarrow \begin{cases} b = -2c \\ b = a-d \\ b = a-d \\ 0 = 0 \end{cases} \Leftrightarrow \begin{cases} b = -2c \\ a = d-2c \end{cases}$$

$$B = \begin{bmatrix} d-2d & -2c \\ c & d \end{bmatrix}, \forall c,d \in \mathbb{R}$$

10. (1º mini-teste 24/04/2013 - grupo II - exc 1)

Discuta em função dos parâmetros reais a e b o sistema:

$$\begin{cases} -ax + ay - z = b-2a \\ x + z = 2 \\ x - 3ay + (a^2+6)z = a-2b+2 \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} -a & a & -1 & b-2a \\ 1 & 0 & 1 & 2 \\ 1 & -3a & a^2+6 & a-2b+2 \end{array}\right]$$

$$\underset{C_2 \leftrightarrow C_1}{\Leftrightarrow} \left[\begin{array}{ccc|c} a & -a & -1 & b-2a \\ 0 & 1 & 1 & 2 \\ -3a & 1 & a^2+6 & a-2b+2 \end{array}\right] \underset{L_3+3L_1}{\Leftrightarrow} \left[\begin{array}{ccc|c} a & -a & -1 & b-2a \\ 0 & 1 & 1 & 2 \\ 0 & 1-3a & a^2+3 & -5a+b+2 \end{array}\right]$$

$$\underset{L_3-(1-3a)L_2}{\Leftrightarrow} \left[\begin{array}{ccc|c} a & -a & -1 & b-2a \\ 0 & 1 & 1 & 2 \\ 0 & 0 & a^2+3a+2 & a+b \end{array}\right]$$

$\Leftrightarrow$. $a \neq 0 \wedge a^2+3a+2 \neq 0$

$\Leftrightarrow a \neq \frac{-3 \pm \sqrt{9-8}}{2}$

$\Leftrightarrow a \neq 1 \wedge a \neq -2$ <u>SPD</u>

. SPD: $a \neq 0 \wedge a \neq -2 \wedge a \neq -1$

SPI: $(a=-2 \wedge b=2) \vee (a=-1 \wedge b=1) \vee (a=0 \wedge b=0)$

. SI: $(a=-2 \wedge b \neq 2) \vee (a=-1 \wedge b \neq 1) \vee (a=0 \wedge b \neq 0)$

//

1. Determine as matrizes de 2ª ordem não nulas que verifiquem

a) $A^2 = I$ $\quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & cb+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Leftrightarrow$

$$\Leftrightarrow \begin{cases} a^2+bc=1 \\ b(a+d)=0 \\ c(a+d)=0 \\ cb+d^2=1 \end{cases} \Leftrightarrow \overset{①}{\begin{cases} a^2=1 \\ b=0 \\ c=0 \\ d^2=1 \end{cases}} \vee \overset{②}{\begin{cases} a^2=1 \\ b=0 \\ d+a=0 \\ d^2=1 \end{cases}} \vee \overset{③}{\begin{cases} a^2=1 \\ a+d=0 \\ c=0 \\ d^2=1 \end{cases}} \vee \overset{④}{\begin{cases} a^2+bc=1 \\ a+d=0 \\ a+d=0 \\ cb+d^2=1 \end{cases}}$$

$$① \begin{cases} a=\pm 1 \\ b=0 \\ c=0 \\ d=\mp 1 \end{cases} \Leftrightarrow \begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix}$$

$$② \begin{cases} a=\pm 1 \\ b=0 \\ d=-a \\ d=\mp 1 \end{cases} \Leftrightarrow \begin{bmatrix} +1 & 0 \\ c & -1 \end{bmatrix} \vee \begin{bmatrix} -1 & 0 \\ c & +1 \end{bmatrix}, \forall c \in \mathbb{R}$$

$$③ \begin{cases} a=\pm 1 \\ d=-a \\ c=0 \\ d=\mp 1 \end{cases} \Leftrightarrow \begin{bmatrix} 1 & b \\ 0 & -1 \end{bmatrix} \vee \begin{bmatrix} -1 & b \\ 0 & 1 \end{bmatrix}$$

$$④ \begin{cases} a^2+bc=1 \\ d=-a \\ 0=0 \\ cb+a^2=1 \end{cases} \Leftrightarrow \begin{cases} b=\frac{1-a^2}{c} \\ d=-a \\ 0=0 \\ cb+a^2=1 \end{cases} \Leftrightarrow \begin{cases} b=\frac{1-a^2}{c}, \ c\neq 0 \\ d=-a \\ 0=0 \\ 0=0 \end{cases} \qquad \begin{bmatrix} a & \frac{1-a^2}{c} \\ c & -a \end{bmatrix}, \forall a \in \mathbb{R} \wedge c \in \mathbb{R}\setminus\{0\}$$

$$\text{se } c=0 \begin{cases} a^2=1 \\ d=-a \\ 0=0 \\ 0=0 \end{cases} \quad \begin{bmatrix} 1 & b \\ 0 & -1 \end{bmatrix} \vee \begin{bmatrix} -1 & b \\ 0 & 1 \end{bmatrix} \quad \underline{\text{já visto}}$$

b) $(2A)^2+I^2=0 \Leftrightarrow 4A^2=-I^2 \Leftrightarrow 4A^2=-I$

$$4\begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & cd+d^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 4a^2+4bc & 4ab+4bd \\ 4ac+4dc & 4cd+4d^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} 4a^2+4bc=-1 \\ 4ab+4bd=0 \\ 4ac+4dc=0 \\ 4cd+4d^2=-1 \end{cases} \Leftrightarrow \begin{cases} 4a^2+4bc=-1 \\ 4b(a+d)=0 \\ 4c(a+d)=0 \\ 4cd+4d^2=-1 \end{cases} \Leftrightarrow \overset{①\ \text{imp.}}{\begin{cases} 4a^2=-1 \\ b=0 \\ c=0 \\ 4d^2=-1 \end{cases}} \vee \overset{②\ \text{imp.}}{\begin{cases} 4a^2=-1 \\ b=0 \\ d=-a \\ 4cd+4d^2=-1 \end{cases}} \vee$$

$$\vee \overset{③\ \text{imp.}}{\begin{cases} a^2=-\frac{1}{4} \\ d=-a \\ c=0 \end{cases}} \vee ④ \begin{cases} a^2+bc=-\frac{1}{4} \\ d=-a \\ d=-a \\ cb+d^2=-\frac{1}{4} \end{cases} \Leftrightarrow \begin{cases} b=\frac{-\frac{1}{4}-a^2}{c}, \ c\neq 0 \\ d=-a \\ 0=0 \\ 0=0 \end{cases} \quad \text{se } c=0 \ \left\{ a^2=-\frac{1}{4} \ \text{imp.} \right.$$

solução $\begin{bmatrix} a & \frac{-1+4a^2}{4c} \\ c & -a \end{bmatrix}$, $\forall a \in \mathbb{R} \wedge c \in \mathbb{R}\setminus\{0\}$

2. Indique a relação que se deve verificar entre os elementos de uma matriz $A = (a_{ij})_{3\times 3}$ para que seja:

a) simétrica $A = A^T$

$$A = \begin{bmatrix} a_{11} & \boxed{a_{12}} & \boxed{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad A^T = \begin{bmatrix} a_{11} & \boxed{a_{21}} & \boxed{a_{31}} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$a_{ij} = a_{ji}, \forall i,j$

exemplo: $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 10 & 0 \\ -1 & 0 & 1000 \end{bmatrix}$ $\qquad a_{12} = a_{21}$, $a_{31} = a_{13}$

b) anti-simétrica $A^T = -A$

$\begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -100 \\ -3 & 100 & 0 \end{bmatrix}$

$a_{ij} = -a_{ji}, \forall i \neq j \wedge a_{ii} = 0$

$a_{ij} = -a_{ji}, \forall i \neq j \wedge a_{ij} = 0, \forall i = j$

3. Resolver em ordem a X as equações matriciais e apresente sempre que possível o resultado como produto de factores, considere A, B, C e X matrizes regulares.

a) $[(A^T)^{-1} \cdot X]^T + (AB)^{-1} = A \Leftrightarrow X^T \cdot [(A^T)^{-1}]^T + B^{-1} \cdot A^{-1} = A \Leftrightarrow X^T \cdot [(A^{-1})^T]^T = A - B^{-1}A^{-1} \Leftrightarrow$
$\Leftrightarrow X^T \cdot A^{-1}A = (A - B^{-1} \cdot A^{-1})A \Leftrightarrow X^T = A^2 - B^{-1}\underbrace{(A^{-1} \cdot A)}_{=I} \Leftrightarrow (X^T)^T = (A^2 - B^{-1})^T \Leftrightarrow X = (A^2 - B^{-1})^T$

b) $XAB + (B^T C X^T)^T = I \Leftrightarrow XAB + XC^T B = I \Leftrightarrow (XA + XC^T) B B^{-1} = I B^{-1} \Leftrightarrow$
$\Leftrightarrow XA + XC^T = B^{-1} \Leftrightarrow X^{-1} \cdot X(A + C^T) = X \cdot B^{-1} \Leftrightarrow (A + C^T)B = X^{-1} \cdot \underbrace{(B^{-1} \cdot B)}_{=I} \Leftrightarrow$
$\Leftrightarrow (X^{-1})^{-1} = [(A + C^T) \cdot B]^{-1} \Leftrightarrow X = B^{-1} \cdot (A + C^T)^{-1}$

c) $(X^T A^{-1} - B)^{-1} = 2AB^{-1} \Leftrightarrow X^T \cdot A^{-1} - B = (2A)^{-1} \cdot B \Leftrightarrow X^T \cdot A^{-1} = (2A)^{-1} \cdot B + B \Leftrightarrow$
$\Leftrightarrow X^T \cdot \underbrace{(A^{-1} \cdot A)}_{=I} = \frac{1}{2} BA^{-1}A + BA \Leftrightarrow X^T = -\frac{1}{2} B + BA = X = (A^T - \frac{I}{2})^{-1} \cdot B^T$

nota! $A^2 + 2A = A(A + 2I)$

d) $[A^{-1} + (B^T)^{-1} X]^T = (BA^T)^{-1} \Leftrightarrow (A^{-1})^T + [(B^T)^{-1} X]^T = (A^T)^{-1} B^{-1} \Leftrightarrow (A^{-1})^T + X^T B^{-1} =$
$= (A^T)^{-1} B^{-1} \Leftrightarrow X^T B^{-1} = (A^T)^{-1} B^{-1} - (A^{-1})^T \Leftrightarrow X^T B^{-1} \cdot B = (A^T)^{-1} B^{-1} \cdot B - (A^{-1})^T B \Leftrightarrow X^T = (A^T)^{-1} -$
$-(A^{-1})^T \cdot B \Leftrightarrow (X^T)^T = [(A^T)^{-1} \cdot (I - B)]^T \Leftrightarrow X = (I - B)^T \cdot A^{-1} \Leftrightarrow X = (I - B^T) \cdot A^{-1}$

e) $[(AX)^T + BC]^{-1} = I \Leftrightarrow ([(AX)^T + BC]^{-1})^{-1} = (I)^{-1} \Leftrightarrow (AX)^T + BC = I^{-1} \Leftrightarrow X^T A^T = I^{-1} - BC \Leftrightarrow$
$\Leftrightarrow (X^T A^T)^T = (I^{-1} - BC)^T \Leftrightarrow A^{-1} A X = A^{-1} (I^{-1} - BC)^T \Leftrightarrow A^{-1} (I - C^T B^T)$

nota! • $I^{-1} = I$ e $I^T = I$ • $(A^T)^{-1} \rightarrow$ inversa da transposta

f) $(A^{-1}X^T - B^T)^{-1} = (A^T B^{-1})^T \Leftrightarrow (A^{-1}X^T - B^T)^{-1} = (B^{-1})^T A \Leftrightarrow (X^T)^{-1}(A^{-1})^T - (B^T)^{-1} = B^{-1T}.A \Leftrightarrow$

$\Leftrightarrow (X^T)^{-1}(A^T)^{-1} = (B^{-1})^T.A + (B^{-1})^T \Leftrightarrow (A)^{-1}(X)^{-1} = A^T.(B)^{-1} + (B)^{-1} \Leftrightarrow A(A)^{-1}X^{-1} = A.A^T.B^{-1} + AB^{-1}$

$\Leftrightarrow X^{-1} = A^{-1}(A^T.B^{-1} + B^{-1})^{-1} \Leftrightarrow X = A^{-1}(B(A^{-1})^T + B)$

g) $(X^{-1}AX)^{-1} + [(A^T)^{-1} - X^T]^T = A^{-1} + AX \Leftrightarrow (AX)^{-1}X + \cancel{A^{-1}} - X = \cancel{A^{-1}} + AX \Leftrightarrow$

$\Leftrightarrow A^{-1}X^{-1}A^{-1}X - A^{-1}X = \underbrace{A^{-1}A}_{I}X \Leftrightarrow A^{-1}X^{-1}A^{-1}X - A^{-1}X = X \Leftrightarrow A^{-1}X^{-1}A^{-1}\underline{X.X^{-1}} - A^{-1}\underline{XX^{-1}} = \underline{XX^{-1}}$

$\Leftrightarrow A^{-1}X^{-1}A^{-1} - A^{-1} = 1 \Leftrightarrow X^{-1}A^{-1} - I = I \Leftrightarrow X^{-1} - A = A \Leftrightarrow X = 2A$

f) $[A^{-1}X^T - B^T]^{-1} = (B^{-1})^T(A^T)^T \Leftrightarrow [[A^{-1}X^T - B^T]^{-1}]^{-1} = ((B^T)^{-1}A)^{-1} \Leftrightarrow A^{-1}X^T - B^T = A^{-1}B^T$

$\Leftrightarrow A^{-1}X^T = A^{-1}B^T + B^T \Leftrightarrow AA^{-1}X^T = A(A^{-1}B^T + B^T) \Leftrightarrow (X^T)^T = (B^T + AB^T)^T \Leftrightarrow X = B + (AB^T)^T$

$\Leftrightarrow X = B + BA^T \Leftrightarrow X = B(I + A^T)$

g) $X^{-1}A^{-1}X + \cancel{A^{-1}} - X = \cancel{A^{-1}} + AX \Leftrightarrow (X^{-1}A^{-1}X - X)X^{-1} = (AX)X^{-1} \Leftrightarrow X^{-1}A^{-1} - I = A \Leftrightarrow X^{-1}A^{-1}A = (A+I)A$

$\Leftrightarrow (X^{-1})^{-1} = [A(A+I)]^{-1} \Leftrightarrow X = (A+I)^{-1}.A^{-1}$

4. Seja B uma matriz simétrica, C uma matriz regular e A uma matriz regular que verificam a seguinte igualdade: $A = B + I$

Resolva em ordem a X a equação:

$$A(C^{-1}X^TC + B)^T = A^2 \Leftrightarrow A^{-1}A(C^{-1}X^TC + B)^T = A^{-1}A^2$$

dados:

$B = B^T$

$C^{-1}C = CC^{-1} = I$

$A^{-1}A = A.A^{-1} = I$

$A = B + I$

transposta do produto é do último → primeiro

$\Leftrightarrow (C^{-1}X^TC)^T + B^T = A^{-1}AA$

$\Leftrightarrow C^TX(C^{-1})^T + B^T = A$

$(C^{-1})^T.C^T = I$

$\Leftrightarrow C^{-1}X^TC = (A-B)^T$

$\Leftrightarrow X^T = C(A-B)^TC^{-1}$

$\Leftrightarrow X^T = C(\cancel{B} + I - \cancel{B})^TC^{-1}$

$\Leftrightarrow X^T = CC^{-1}$

$\Leftrightarrow X = I^T \Leftrightarrow X = I$

5. Mostre que se A é idempotente então $A^m = A$ para todo o $m \in \mathbb{N}$

$A^2 = A$

$A^3 = A^2.A = A.A = A^2 = A$

$A^4 = A^3.A = A.A = A^2 = A$ ... $A^m = A^{m-1}.A = A.A = A^2 = A$ c.q.d.

6. Mostre que se A é uma matriz quadrada de ordem m, simétrica e se P é uma matriz $m \times n$ então $B = P^TAP$ também é simétrica (repita para anti-simétrica)

$A_{m \times m}$ $A^T = A$

$P_{m \times n}$ $B = P^TAP$

$B = B^T \rightarrow P^TAP = (P^TAP)^T \Leftrightarrow P^TAP = P^TA^TP$

$A^T = A$

$P^TAP = P^TAP$ c.q.d.

verificar se é quadrada

$[B] = [P^T]_{n \times m}[A]_{m \times m}[P]_{m \times n} = [P^TA]_{n \times m}.[P]_{m \times n} = [B] = [P^T]_{n \times m}[A]_{m \times m}[P]_{m \times n}$

$= [P^TA]_{n \times m}[P]_{m \times n} = [P^TAP]_{n \times n}$

7. Mostre que o produto de matrizes ortogonais é uma matriz ortogonal

↳ nota! uma matriz quadrada diz-se ortogonal se • A é invertível e $A^T = A^{-1}$

ou

• $A^TA = AA^T = I$

seja A uma matriz ortogonal : $A^TA = AA^T = I$

seja B uma matriz ortogonal $B^TB = BB^T = I$

$M = A.B$

$$MM^T = I \iff (AB)(AB)^T = I \iff A\underbrace{BB^T}_{I}A^T = I \iff AA^T = I \iff I = I \quad \text{c.q.d.}$$

8. (1º mini-teste 2º semestre ex 4. gII (6/03/2012))

Se A é ortogonal e se $B = AC$ é regular, então CB^{-1} é ortogonal

A é ortogonal se $AA^T = A^TA = I$

$B = A.C$

$BB^{-1} = B^{-1}B = I \rightarrow$ regular

$$\cdot\ (CB^{-1})(CB^{-1})^T = I$$

$$\iff CB^{-1}.(B^{-1})^T.C^T = I$$

$$\iff C(AC)^{-1}.[(AC)^{-1}]^T C^T = I$$

$$\iff \underbrace{C.C^{-1}}_{I}A^{-1}.(C^{-1}A^{-1})^T.C^T = I$$

$$\iff A^{-1}.(A^{-1})^T.\underbrace{(C^T)^{-1}C^T}_{I} = I$$

$$\iff A^{-1}(A^{-1})^T = I$$

$$\iff (A^T.A)^{-1} = I$$

$$\iff I^{-1} = I \iff I = I \quad \text{c.q.d.}$$

9. Encontre a e b de forma que A seja a matriz inversa de B, onde:

$$A = \begin{bmatrix} 2 & -1 & -1 \\ a & \frac{1}{4} & b \\ \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \end{bmatrix} \quad e \quad B = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 6 \\ 1 & 3 & 2 \end{bmatrix}$$

$A.B = I$

$$\begin{bmatrix} 2 & -1 & -1 \\ a & \frac{1}{4} & b \\ \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 6 \\ 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-1-3 & 8-6-2 \\ a+b & 2a+\frac{1}{4}+3b & 4a+\frac{6}{4}+2b \\ \frac{1}{8}-\frac{1}{8} & \frac{2}{8}+\frac{1}{8}-\frac{3}{8} & \frac{4}{8}+\frac{6}{8}-\frac{2}{8} \end{bmatrix}$$

$$\iff \begin{bmatrix} 1 & 0 & 0 \\ a+b & 2a+3b+\frac{1}{4} & 4a+2b+\frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \iff \begin{cases} a+b=0 \\ 2a+3b+\frac{1}{4}=1 \\ 4a+2b+\frac{3}{2}=0 \end{cases} \iff$$

$$\iff \begin{cases} a=-b \\ -2b+3b=\frac{3}{4} \\ \text{—} \end{cases} \iff \begin{cases} -a=+b \\ b=\frac{3}{4} \\ a=-\frac{3}{4} \end{cases} //$$

Inversa por condensação

1. Determine, por condensação, a matriz inversa das matrizes

a) $A = \begin{bmatrix} 5 & -4 \\ -8 & 6 \end{bmatrix}$ b) $B = \begin{bmatrix} -2 & 3 & 1 \\ 3 & 6 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 \\ -3 & -2 & 1 \\ 4 & -16 & 8 \end{bmatrix}$

$\begin{bmatrix} -3 & -2 \\ -4 & -\frac{5}{2} \end{bmatrix}$

b)

$$\left[\begin{array}{ccc|ccc} -2 & 3 & 1 & 1 & 0 & 0 \\ 3 & 6 & 2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \underset{L_1 \leftrightarrow L_3}{\Leftrightarrow} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 3 & 6 & 2 & 0 & 1 & 0 \\ -2 & 3 & 1 & 1 & 0 & 0 \end{array}\right] \underset{\substack{L_2-3L_1 \\ L_3+2L_1}}{\Leftrightarrow} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & -3 \\ 0 & 7 & 3 & 1 & 0 & 2 \end{array}\right] \Leftrightarrow$$

$$\underset{\Leftrightarrow}{L_2 \leftrightarrow L_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & 7 & 3 & 1 & 0 & 2 \\ 0 & 0 & -1 & 0 & 1 & -3 \end{array}\right] \underset{L_2+3L_3}{\overset{L_1+L_3}{\Leftrightarrow}} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 1 & -2 \\ 0 & 7 & 0 & 1 & 3 & -7 \\ 0 & 0 & -1 & 0 & 1 & -3 \end{array}\right] \underset{L_3 \times (-1)}{\overset{L_2 : 7}{\Leftrightarrow}} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 1 & -2 \\ 0 & 1 & 0 & \frac{1}{7} & \frac{3}{7} & -1 \\ 0 & 0 & 1 & 0 & -1 & 3 \end{array}\right]$$

$$\underset{\Leftrightarrow}{L_1 - 2L_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{2}{7} & \frac{1}{7} & 0 \\ 0 & 1 & 0 & \frac{1}{7} & \frac{3}{7} & -1 \\ 0 & 0 & 1 & 0 & -1 & 3 \end{array}\right] \qquad B^{-1} = \begin{bmatrix} -\frac{2}{7} & \frac{1}{7} & 0 \\ \frac{1}{7} & \frac{3}{7} & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

c)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -3 & -2 & 1 & 0 & 1 & 0 \\ 4 & -16 & 8 & 0 & 0 & 1 \end{array}\right] \overset{L_3 : 4}{\Leftrightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -3 & -2 & 1 & 0 & 1 & 0 \\ 1 & -4 & 2 & 0 & 0 & \frac{1}{4} \end{array}\right] \underset{L_3-L_1}{\overset{L_2+3L_1}{\Leftrightarrow}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 3 & 1 & 0 \\ 0 & -4 & 2 & -1 & 0 & \frac{1}{4} \end{array}\right]$$

$$\underset{\Leftrightarrow}{L_3 - L_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & -7 & -2 & \frac{1}{4} \end{array}\right]$$

não tem inversa (C é uma matriz singular) $|C| = 0$

d) $D = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} & 1 \\ 1 & 0 & -1 \\ \frac{1}{3} & \frac{2}{3} & -1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \underset{2L_2 - L_1}{\overset{L_3 - L_1}{\Leftrightarrow}} \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & 3 & -2 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{array}\right] \Leftrightarrow$$

$$\underset{\Leftrightarrow}{3L_3 - L_2} \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & 3 & -2 & 1 & 0 \\ 0 & 0 & -3 & -1 & -1 & 3 \end{array}\right] \underset{L_2+L_3}{\overset{L_1}{\Leftrightarrow}} \left[\begin{array}{ccc|ccc} & & & & & \\ 0 & -3 & 0 & -3 & 0 & 3 \\ 0 & 0 & -3 & -1 & -1 & 3 \end{array}\right]$$

3. Calcule a solução do sistema, sem o resolver:
$$\begin{cases} 2x+y+z=3 \\ x-y+2z=-3 \\ 2x+z=0 \end{cases}$$

$Dx = B \Leftrightarrow x = D^{-1}.B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \text{inversa } D \end{bmatrix} \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$$

Determinantes

1. Calcule os seguintes determinantes

a) $\begin{vmatrix} 3^t & 2^t \\ 3^{t-1} & 2^{t-1} \end{vmatrix} = 3^t.2^{t-1} - 2^t.3^{t-1} = 3^t.\frac{2^t}{2} - \frac{3^t}{3}.2^t = \frac{6^t}{2} - \frac{6^t}{3} = 6^t\left(\frac{1}{2}-\frac{1}{3}\right)$

$= 6^t.\frac{1}{6} = \frac{6^t}{6} = 6^{t-1}$

b) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 0 & 0 \end{vmatrix} \begin{matrix} 1 & -1 \\ 1 & 3 \\ 1 & 0 \end{matrix} = -2$ regra sarrus

$\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ \boxed{1 \; 0 \; 0} \end{vmatrix} = (1).\overbrace{(-1)^{3+1}}^{(-1)^{i+j}}.\begin{vmatrix} -1 & 0 \\ 3 & 2 \end{vmatrix} = -2-0 = -2$

c) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix} \overset{C_2+C_1}{=} \begin{vmatrix} \boxed{1 \; 0 \; 0} \\ 1 & 4 & 2 \\ 1 & 3 & 1 \end{vmatrix} = (1).(-1)^2.\begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix} = -2$

d) $\begin{vmatrix} 2 & 1 & 3 & 9 \\ -1 & 2 & 1 & 0 \\ 2 & -2 & 1 & 3 \\ \boxed{1 \; 0 \; 1 \; -2} \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 & 9 \\ -1 & 2 & 2 & -2 \\ 2 & -2 & -1 & 7 \\ \boxed{1 \; 0 \; 0 \; 0} \end{vmatrix} = (1)(-1)^5 \begin{vmatrix} 1 & 1 & 9 \\ 2 & 2 & -2 \\ -2 & -1 & 7 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & 9 \\ 2 & 0 & -2 \\ -2 & 1 & 7 \end{vmatrix} =$

$C_3-C_1 \quad C_4+2C_1 \qquad\qquad C_2-C_1$

$= -(1)(-1)^5 \begin{vmatrix} 1 & 9 \\ 2 & -2 \end{vmatrix} = -2-18 = -20$

e) $\begin{vmatrix} 1 & 1 & 2 & 1 & 0 \\ 2 & 0 & 1 & 2 & 1 \\ -1 & 1 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & -1 \\ 0 & -1 & 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 & 1 & 0 \\ 2 & 0 & 1 & 2 & 1 \\ -1 & 0 & 2 & 1 & 3 \\ 3 & 0 & 1 & 2 & -1 \\ 0 & -1 & 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \cancel{1} & \cancel{1} & \cancel{2} & \cancel{1} & \cancel{0} \\ 2 & 0 & 1 & 2 & 1 \\ -1 & 0 & 2 & 1 & 3 \\ 3 & 0 & 1 & 2 & -1 \\ 1 & 0 & 4 & 1 & 1 \end{vmatrix} =$

$L_3 + L_5$ $\qquad$ $L_5 + L_1$

$= (1).(-1)^3 \begin{vmatrix} 2 & 1 & 2 & 1 \\ -1 & 2 & 1 & 3 \\ 3 & 1 & 2 & -1 \\ 1 & 4 & 1 & 1 \end{vmatrix} = - \begin{vmatrix} 2 & 1 & 0 & 1 \\ -1 & 2 & 2 & 3 \\ 3 & 1 & -1 & -1 \\ 1 & 4 & 0 & 1 \end{vmatrix} = - \begin{vmatrix} 2 & 1 & 0 & 1 \\ +5 & 4 & 0 & 1 \\ \cancel{3} & \cancel{1} & \cancel{-1} & \cancel{-1} \\ 1 & 4 & 0 & 1 \end{vmatrix} =$

$C_3 + C_1$ $\qquad$ $L_2 + 2L_3$

$= -(-1)^6 . \begin{vmatrix} 2 & 1 & 1 \\ -5 & 4 & 1 \\ 1 & 4 & 1 \end{vmatrix} \begin{matrix} 2 & 1 \\ -5 & 4 \\ 1 & 4 \end{matrix} = -(\cancel{8}+1+20+5-\cancel{8}-4) = 18$

f) ex 1, g II, 1ª freq. 23/10/2013

1. calcule o determinante da matriz

$c_1 : a \quad c_2 : a \quad c_3 : a$

$A = \begin{bmatrix} a & \boxed{a} & 2a & a & 0 \\ 2a & 0 & a & 2a & a \\ -a & a & 0 & a & 2a \\ 3a & 0 & a & 2a & -a \\ 0 & -a & 2a & 0 & a \end{bmatrix} = \begin{vmatrix} a & a & 2a & a & 0 \\ 2a & 0 & a & 2a & a \\ -a & a & 0 & a & 2a \\ 3a & 0 & a & 2a & -a \\ -a & 0 & 2a & a & 3a \end{vmatrix} = \begin{vmatrix} \cancel{a} & \cancel{a} & \cancel{2a} & \cancel{a} & \cancel{0} \\ 2a & 0 & a & 2a & a \\ -2a & 0 & -2a & 0 & 2a \\ 3a & 0 & a & 2a & -a \\ -a & 0 & 2a & a & 3a \end{vmatrix} =$

$L_5 + L_3$ $\qquad$ $L_3 - L_1$

$12a^5$

$= (a)(-1)^3 \begin{vmatrix} 2a & a & 2a & a \\ -2a & -2a & 0 & 2a \\ 3a & a & 2a & -a \\ -a & 2a & a & 3a \end{vmatrix} = -a \begin{vmatrix} 2a & a & 2a & a \\ -2a & -2a & 0 & 2a \\ a & 0 & 0 & -2a \\ -a & 2a & a & 3a \end{vmatrix} =$

$L_3 - L_1$

1º por em evidência

3. Calcule os determinantes

a)
$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} \overset{C_3+C_2}{=} \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix} \overset{C_3 : (a+b+c)}{=} (a+b+c) \underbrace{\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}}_{=0 \text{, pois } C_1 = C_3} = 0$$

b)
$$\begin{vmatrix} x-y & x-y & x^2-y^2 \\ 1 & 1 & x+y \\ y & 1 & x \end{vmatrix} = (x-y) \cdot \underbrace{\begin{vmatrix} 1 & 1 & x+y \\ 1 & 1 & x+y \\ y & 1 & x \end{vmatrix}}_{=0 \rightarrow L_1 = L_2}$$

c)
$$\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & x^3 \\ x^2+1 & x^3-1 & x^4+5 \end{vmatrix} = 0 \text{, pois } L_2 = xL_1 \text{ (proporcionais)}$$

d)
$$\begin{vmatrix} b^2-ab & b-c & ac-bc \\ a^2-ab & b-a & b^2-ab \\ bc-ac & c-a & a^2-ab \end{vmatrix} = \begin{vmatrix} b(b-a) & b-c & c(a-b) \\ -a(-a+b) & b-a & -b(-b+a) \\ c(b-a) & c-a & a(a-b) \end{vmatrix} = (b-a)(a-b) \begin{vmatrix} b & b-c & c \\ -a & b-a & -b \\ c & c-a & a \end{vmatrix} =$$

$C_1 : (b-a)$ $\quad C_3 (a-b)$

$$= (b-a)(a-b) \begin{vmatrix} b & b & c \\ -a & -a & -b \\ c & c & a \end{vmatrix} = 0 \text{, pois } C_1 = C_2$$

e)
$$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = \begin{vmatrix} 2x+2y & y & x+y \\ 2x+2y & x+y & x \\ 2x+2y & x & y \end{vmatrix} = 2(x+y) \begin{vmatrix} 1 & x & x+y \\ 1 & x+y & x \\ 1 & x & y \end{vmatrix} \overset{\substack{L_2-L_1 \\ L_3-L_1}}{=}$$

$C_1 \rightarrow C_1 + C_2 + C_3$

$$= 2(x+y) \begin{vmatrix} 1 & y & x+y \\ 0 & x & -y \\ 0 & x-y & -x \end{vmatrix} = 2(x+y)(1)(-1)^2 \cdot \begin{vmatrix} x & -y \\ x-y & -x \end{vmatrix} = 2(x+y) \cdot [-x^2 + y(x-y)]$$

4. Resolva a equação:

$$\begin{vmatrix} x^2+x & 3x & 2x \\ x^2-1 & x-1 & 3x-3 \\ (x+1)^2 & x & x-2 \end{vmatrix} = \begin{vmatrix} x(x+1) & 3 \cdot x & 2 \cdot x \\ (x-1)(x+1) & x-1 & (x-1)3 \\ (x+1)^2 & x & x-2 \end{vmatrix} = x \cdot (x-1) \begin{vmatrix} x+1 & 3 & 2 \\ x+1 & 1 & 3 \\ (x+1)^2 & x & x-2 \end{vmatrix} =$$

$C_1 : (x+1)$

$= n(n-1)(n+1) \begin{vmatrix} 1 & 3 & 2 \\ 1 & 1 & 3 \\ 1 & n & n-2 \end{vmatrix} \underset{L_3-L_1}{\overset{L_2-L_1}{=}} \begin{vmatrix} 1 & 3 & 2 \\ 0 & -2 & 1 \\ 0 & n-3 & n-4 \end{vmatrix} = n(n-1)(n+1)(-1)^2 \begin{vmatrix} -2 & 1 \\ n-3 & n-4 \end{vmatrix} =$

$= n.(n^2-1)(-2n+8-n+3) = n.(n^2-1)(n^2-1)(-3n+11) = n=0 \vee n=1 \vee n=-1 \vee n=\frac{-11}{3}$

5. Calcule os determinantes, apresentando o resultado sob a forma de um produto de factores do 1º grau.

a) $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} \underset{C_1 \to C_1+C_2+C_3}{=} \begin{vmatrix} a & -c & -b \\ b & a+b+c & -a \\ c & -a & a+b+c \end{vmatrix} \overset{C_3+C_2}{=} \begin{vmatrix} -a & +c & +b+c \\ b & a+b+c & b+c \\ c & -a & b+c \end{vmatrix} =$

$= -(b+c) \begin{vmatrix} -a & +c & 1 \\ b & a+b+c & 1 \\ c & -a & 1 \end{vmatrix} \underset{L_3-L_1}{\overset{L_2-L_1}{=}} \begin{vmatrix} -a & +c & 1 \\ b-a & a+b & 0 \\ c+a & -a-c & 0 \end{vmatrix} = (b+c)(-1)^4 \begin{vmatrix} b+a & -a-b \\ c+a & -a+c \end{vmatrix} =$

$= (b+c)(c+a) \begin{vmatrix} b-a & -a-b \\ 1 & 1 \end{vmatrix} = (b+c)(c+a)(b+a+a+b) = (b+c)(c+a)(2b+2a)$

b) $\begin{vmatrix} n-4 & 1 & 1 & 1 \\ 4 & n-1 & -1 & -3 \\ 5 & -2 & n-3 & -1 \\ -6 & 1 & 2 & n+2 \end{vmatrix} \underset{C_1 \to C_1+C_2+C_3+C_4}{=} \begin{vmatrix} n-1 & 1 & 1 & 1 \\ n-1 & n-1 & -1 & -3 \\ n-1 & -2 & n-3 & -1 \\ n-1 & 1 & 2 & n+2 \end{vmatrix} = (n-1) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & n-1 & -1 & -3 \\ 1 & -2 & n-3 & -1 \\ 1 & 1 & 2 & n+2 \end{vmatrix} =$

$\underset{L_4-L_1}{\overset{L_2-L_1,\ L_3-L_1}{=}} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & n-2 & -2 & -4 \\ 0 & -3 & n-4 & -2 \\ 0 & 0 & 1 & n+1 \end{vmatrix} = (n-1) \begin{vmatrix} n-2 & -2 & -4 \\ -3 & n-4 & -2 \\ 0 & 1 & n+1 \end{vmatrix} = (n-1) \overset{L_1 \to L_1+L_2+L_3}{=} (n-1) \begin{vmatrix} n-5 & n-5 & n-5 \\ -3 & n-4 & -2 \\ 0 & 1 & n+1 \end{vmatrix} =$

$= (n-1)(n-5) \begin{vmatrix} 1 & 1 & 1 \\ -3 & n-4 & -2 \\ 0 & 1 & n+1 \end{vmatrix} \overset{L_2+3L_1}{=} (n-1)(n-5) \begin{vmatrix} 1 & 1 & 1 \\ 0 & n-1 & 1 \\ 0 & 1 & n+1 \end{vmatrix} = (n-1)(n-5) \begin{vmatrix} n-1 & 1 \\ 1 & n+1 \end{vmatrix} =$

$= (n-1)(n-5)(n^2-2) = (n-1)(n-5)(n-\sqrt{2})(n+\sqrt{2})$

nota: A regular $\xrightarrow{\text{logo}}$ • admite inversa e • $|A| \neq 0$

6. Use a regra de Cramer para resolver o seguinte sistema de equações em ordem a x_1, x_2 e x_3

$$\begin{cases} x_1 - x_2 + x_3 = 2 \\ x_1 + x_2 - x_3 = 0 \\ -x_1 - x_2 - x_3 = 6 \end{cases} \qquad \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & -1 & -1 \end{vmatrix} \begin{matrix} 1 & -1 \\ 1 & 1 \\ -1 & -1 \end{matrix} = -1-1-1-1-\cancel{1}+\cancel{1} = -4 \neq 0$$

$$x_1 = \frac{1}{|A|} \cdot \begin{vmatrix} 2 & -1 & 1 \\ 0 & 1 & -1 \\ 6 & -1 & -1 \end{vmatrix} = \frac{-4}{-4} = 1 \qquad x_2 = \frac{1}{|A|} \cdot \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \\ -1 & 6 & -1 \end{vmatrix} = \frac{-16}{4} = -4 \qquad x_3 = \frac{1}{|A|} \begin{vmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \\ -1 & -1 & 6 \end{vmatrix} = \frac{-12}{4} = -3$$

1º mini-teste 6/03/12, grupo I

1. dado o sistema

$$\begin{cases} x+y+2z+w = a \\ x+2y+3z+4w = b \\ 2x+3y+6z+5w = c \\ -x+y+2z+kw = d \end{cases}$$

indique para que valores de k é possível resolver o sistema pela regra de Cramer com a, b, c, d e $k \in \mathbb{R}$

$$|A| \neq 0 \qquad \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 6 & 5 \\ -1 & 1 & 2 & k \end{vmatrix} \underset{L_4+L_1}{\overset{L_2-L_1,\ L_3-2L_1}{=}} \begin{vmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & k+1 \end{vmatrix} = (1)(-1)^2 \begin{vmatrix} 1 & 1 & 3 \\ 1 & 2 & 3 \\ 2 & 4 & k+1 \end{vmatrix} \overset{L_2-L_1}{=} \begin{vmatrix} 1 & 1 & 3 \\ 0 & 1 & 0 \\ 2 & 4 & k+1 \end{vmatrix}$$

$$= (1)(-1)^4 \cdot \begin{vmatrix} 1 & 3 \\ 2 & k+1 \end{vmatrix} = k+1-6 = k-5 \qquad |A| \neq 0 \Leftrightarrow k \neq 5$$

2. Calcule o determinante, apresentando o resultado sob a forma de um produto de fatores em ordem a a e b $\in \mathbb{R}$

$$\begin{vmatrix} b-4a & 2b+6a & 2a \\ a-6b & 2a+4b & 3b \\ 2b & 4a+b & a \end{vmatrix} \underset{C_1+2C_3}{\overset{C_2-3C_3}{=}} \begin{vmatrix} b & 2b & 2a \\ a & 2a & 3b \\ 2b+2a & a+b & a \end{vmatrix} \overset{C_2-2C_1}{=} \begin{vmatrix} b & 0 & 2a \\ a & 0 & 3b \\ 2b+2a & -3a-3b & 0 \end{vmatrix} =$$

$$-3(a+b)(-1)^5 \cdot \begin{vmatrix} b & 2a \\ a & 3b \end{vmatrix} = 3(a+b)(3b^2-2a^2)$$

3. sendo $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 10$ calcule $\begin{vmatrix} 3f-2c & i & 4c \\ 4e-6b & 3h & 12b \\ 6d-4a+3e-2b & 2g+h & 8a+4b \end{vmatrix} \overset{L_2:3,\ C_3:4}{=} 12 \begin{vmatrix} 3f-2c & i & c \\ 3e-2b & h & b \\ 6d-4a+3e-2b & 2g+h & 2a+b \end{vmatrix}$

$$\overset{L_3-L_2}{=} 12 \begin{vmatrix} 3f-2c & i & c \\ 3e-2b & h & b \\ 6d-4a & 2g & 2a \end{vmatrix}_{C_1+2C_3} = 12 \begin{vmatrix} 3f & i & c \\ 3e & h & b \\ 6d & 2g & 2a \end{vmatrix}_{C_1:3} \overset{L_3:2}{=} 72 \begin{vmatrix} f & i & c \\ e & h & b \\ d & g & a \end{vmatrix} \overset{T}{=} 72 \begin{vmatrix} f & e & d \\ i & h & g \\ c & b & a \end{vmatrix} \overset{L_1 \leftrightarrow L_3}{=} -72 \begin{vmatrix} c & b & a \\ i & h & g \\ f & e & d \end{vmatrix} =$$

$$\overset{C_1 \leftrightarrow C_3}{=} +72 \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} \overset{L_2 \leftrightarrow L_3}{=} -72 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -72 \times 10 = -720$$

4. Calcule a inversa da matriz A pela adjunta

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{bmatrix} \qquad |A| = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{vmatrix} \begin{matrix} 2 & 1 \\ 3 & 1 \\ 1 & 0 \end{matrix} = \cancel{2} - \cancel{2} - 3 + 1 = -2 \neq 0$$

admite inversa

$$\text{adj}(A) = \begin{bmatrix} (-1)^2 \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} & (-1)^3 \begin{vmatrix} & \\ & \end{vmatrix} & (-1)^4 \begin{vmatrix} & \\ & \end{vmatrix} \\ (-1)^3 \begin{vmatrix} & \\ & \end{vmatrix} & (-1)^4 \begin{vmatrix} & \\ & \end{vmatrix} & (-1)^5 \begin{vmatrix} & \\ & \end{vmatrix} \\ (-1)^4 \begin{vmatrix} & \\ & \end{vmatrix} & (-1)^5 \begin{vmatrix} & \\ & \end{vmatrix} & (-1)^6 \begin{vmatrix} & \\ & \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} 1 & -5 & -1 \\ -1 & 3 & +1 \\ -1 & +1 & -1 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & -1 \\ -5 & 3 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A = -\frac{1}{2} \begin{bmatrix} 1 & -1 & -1 \\ -5 & 3 & 1 \\ -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{5}{2} & -\frac{3}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Revisões

1. Sendo $(A+I)$ uma matriz regular, prove que $(A+I)^{-1}$ é permutável com $I-A$

$(A+I)^{-1}(I-A) = (I-A)(A+I)^{-1} \Leftrightarrow (A+I)(A+I)^{-1}.(I-A) = (A+I)(I-A)(A+I)^{-1}$

$\Leftrightarrow (I-A)(A+I) = (A+I)(I-A)\underbrace{(A+I)^{-1}(A+I)}_{I} \Leftrightarrow (I-A)(A+I) = (A+I)(I-A)$

$\cancel{A} + I - A^2 - A = A - A^2 + I - \cancel{A}$

$\Leftrightarrow I - A^2 = -A^2 + I \Leftrightarrow I - A^2 = I - A^2$ c.q.d //

2. Sejam A, B, C e D matrizes quadradas de ordem n. Calcule o determinante da matriz $(C^{-1}B)^{-1} + B^{-1}A.C$, sabendo que $C = B^T$, $A = 2D - I$ com $|D| = 4$

$= B^{-1}.C + B^{-1}.A.C = B^{-1}(C + AC) = B^{-1}[B^T + (2D-I)B^T] = B^{-1}(\cancel{B^T} + 2DB^T - \cancel{B^T})$

$= |B^{-1}.2DC| = |B^{-1}|.|2D|.|C| = \frac{1}{\cancel{|B|}}.2^m|D|.\underbrace{\cancel{|B^T|}}_{=|B|} = 2^m|D| = 2^m \times 4$

$= 2^m \times 2^2 = 2^{m+2}$ //

3. Sejam A, B, C e D matrizes quadradas de ordem 5, tais que $C^TAB^{-3}D = 2A$. Sabendo que A é regular, $|C| = 2$ e que a matriz D se obtém de C trocando a 1ª coluna com a última, calcule $|B|$ $\quad |C^TAB^{-3}D| = |2A| \Leftrightarrow |C^T|\cancel{|A|}|B^{-3}|.|D| = 2^5\cancel{|A|}$, pois $|A| \neq 0$

$\Leftrightarrow |C|.\frac{1}{|B|^3}.(-2) = 2^5 \Leftrightarrow 2.\frac{1}{|B|^3}(-2) = 2^5 \Leftrightarrow \frac{1}{|B|^3} = -2^3 \Leftrightarrow |B|^3 = -\frac{1}{2^3} \Leftrightarrow |B| = \sqrt[3]{-\frac{1}{2^3}}$

$\Leftrightarrow |B| = -\frac{1}{2}$

by Staples

Funções

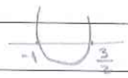
1. Indique, sob a forma de intervalos de nºs reais, o conjunto solução das condições:

a) $x \geq \frac{x^2-3}{1-x} \Leftrightarrow \frac{x^2-3}{1-x} - \underset{(1-x)}{x} \leq 0 \Leftrightarrow \frac{x^2-3-x+x^2}{1-x} \leq 0 \Leftrightarrow \frac{2x^2-x-3}{1-x} \leq 0$

calc. aux.

· $x = \frac{1 \pm \sqrt{1-4\times 2\times(-3)}}{4} \Leftrightarrow x = \frac{1\pm\sqrt{25}}{4} \Leftrightarrow x = \frac{6}{4} = \frac{3}{2} \vee x = -1$ · $1-x = 0 \Leftrightarrow x = 1$

x	$-\infty$	-1		1		$\frac{3}{2}$	$+\infty$
$2x^2-x-3$	+	0	−	−	−	0	+
$1-x$	+	+	+	0	−	−	−
Q	+	0	−	nd	+	0	−



$S = [-1;1[\cup [\frac{3}{2};+\infty[$

b) $\underbrace{|x|+|-x+3|}_{f(x)} > 3$

→ truque: calcular zeros e se for $(-x)$ ← ramo superior ≤, ramo inferior >

$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$ $|-x+3| = \begin{cases} -x+3 & \text{se } -x+3 \geq 0 \\ x-3 & \text{se } -x+3 < 0 \end{cases} = \begin{cases} -x+3 & \text{se } x \leq 3 \\ x-3 & \text{se } x > 3 \end{cases}$

	$-\infty$	0		3	$+\infty$
$\|x\|$	$-x$	$\underset{=0}{x}$	x	$\underset{=3}{x}$	x
$\|-x+3\|$	$-x+3$	$\underset{=3}{-x+3}$	$-x+3$	$\underset{=0}{-x+3}$	$x-3$
$f(x)$	$-2x+3$	3	3	3	$2x-3$

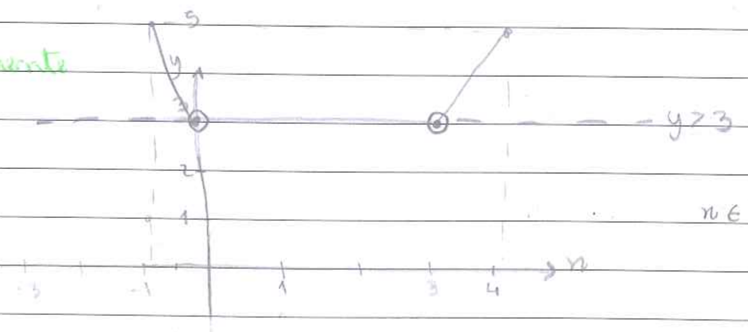
$f(x) = \begin{cases} -2x+3 & \text{se } x < 0 \\ 3 & \text{se } 0 \leq x \leq 3 \\ 2x-3 & \text{se } x > 3 \end{cases}$

$f(x) > 3$ $(-2x+3>3 \wedge x<0) \vee \underset{\text{imp.}}{(3>3 \wedge 0 \leq x \leq 3)} \vee (2x-3>3 \wedge x>3)$

$\Leftrightarrow (-2x>0 \wedge x<0) \vee (2x>6 \wedge x>3)$

$\Leftrightarrow (x<0 \wedge x<0) \vee (x>3 \wedge x>3)$ $x \in]-\infty;0[\cup]3;+\infty[$

b₁) resolva graficamente



$x \in]-\infty;0[\cup]3;+\infty[$

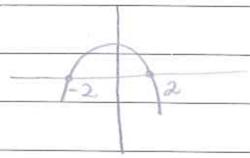
2. Considere a função $f(x) = 2 - \sqrt{4-x^2}$

a) calcule o domínio de f

$D = \{x \in \mathbb{R} : 4-x^2 \geq 0\} \Leftrightarrow [-2;2]$

$x^2 \leq 4$. calc. aux

$x \leq 2 \vee x \geq -2$



calc. aux. $4-x^2=0 \Leftrightarrow x = \pm 2$

b) justifique que f não tem inversa

· como, por exemplo, $f(2) = f(-2) = 2$ então a função não é injetiva, logo não admite inversa

pq se elevou ao quadrado

· $y = 2 - \sqrt{4-x^2} \Leftrightarrow 2 - y = \sqrt{4-x^2} \Rightarrow (2-y)^2 = 4 - x^2 \Leftrightarrow 4 - 4y + y^2 = 4 - x^2 \Leftrightarrow x^2 = 4y - y^2$

$\Leftrightarrow x = \pm\sqrt{4y - y^2}$

não tem inversa

c) calcule o contradomínio

$4y - y^2 \geq 0 \land 2 - y \geq 0$

$\Leftrightarrow y(4-y) \geq 0 \land -y \geq -2$

$\Leftrightarrow 0 \leq y \leq 4 \land y \leq 2$ $\quad y \in [0;4] \cap [-\infty;2]$

$y \in [0;2]$ $\quad CD_f = [0;2]$

ou

$-2 \leq x \leq 2 \Leftrightarrow 0 \leq x^2 \leq 4 \Leftrightarrow 0 \geq -x^2 \geq -4 \Leftrightarrow -4 \leq -x^2 \leq 0 \Leftrightarrow 0 \leq 4 - x^2 \leq 4$

$\Leftrightarrow 0 \leq \sqrt{4-x^2} \leq 2 \Leftrightarrow 0 \geq -\sqrt{4-x^2} \geq -2 \Leftrightarrow -2 \leq -\sqrt{4-x^2} \leq 0$

$\Leftrightarrow 0 \leq 2 - \sqrt{4-x^2} \leq 2$ $\quad CD_f = [0;2]$

· $f(x) = 1 - \sqrt{1 - \ln x}$

a) $D_f = ?$ $\quad D_f = \{x \in \mathbb{R} : 1 - \ln x \geq 0 \land x > 0\} =]0;e]$

$-\ln x \geq -1$

$\ln x \leq 1$

$x \leq e$

b) caracterize a função inversa

$y = 1 - \sqrt{1 - \ln x} \Leftrightarrow 1 - y = \sqrt{1 - \ln x} \underset{1-y \geq 0}{\Rightarrow} (1-y)^2 = 1 - \ln x \Leftrightarrow 1 - 2y + y^2 = 1 - \ln x \Leftrightarrow$

$\Leftrightarrow 2y - y^2 = \ln x \Leftrightarrow e^{2y - y^2} = x$ $\qquad$ · $1 - y \geq 0 \Leftrightarrow -y \geq -1 \Leftrightarrow y \leq 1$

$f^{-1}(x) = e^{2x - x^2}$

$D_{f^{-1}}: \mathbb{R} \cap]-\infty;1] =]-\infty;1]$

caracterização:

$f^{-1}:]-\infty;1] \longrightarrow]0;e]$

$x \longmapsto e^{2x - x^2}$

c) calcule o contradomínio sem a inversa

1º domínio $\rightarrow$ $\quad 0 < x \leq e \Rightarrow \ln x \leq 1 \Leftrightarrow -\ln x \geq -1 \Leftrightarrow 1 - \ln x \geq 0 \Leftrightarrow \sqrt{1 - \ln x} \geq 0$

$\Leftrightarrow -\sqrt{1 - \ln x} \leq 0 \Leftrightarrow \underbrace{1 - \sqrt{1 - \ln x}}_{f(x)} \leq 1$

$\ln(0) < \ln x \leq \ln e$

$\underset{-\infty}{0^+} < \ln x < 1$

$CD_f =]-\infty;1]$

① Caracterize, caso existam, as inversas das funções

a) $f(x) = \pi - 5\arccos(3x-1)$

$\cdot\ D_f = \{x \in \mathbb{R} : \underbrace{-1 \leq 3x-1 \leq 1}_{\text{arccosx}}\} = [0; \frac{2}{3}]$

$0 \leq 3x \leq 2$

$0 \leq x \leq \frac{2}{3}$

$\cdot\ y = \pi - 5\arccos(3x-1) \Leftrightarrow \frac{\pi - y}{5} = \arccos(3x-1) \Leftrightarrow \cos\left(\frac{\pi-y}{5}\right) = 3x-1 \Leftrightarrow \frac{1}{3}\left[1+\cos\left(\frac{\pi-y}{5}\right)\right] = x$

$\cdot\ f^{-1}(x) = \dfrac{1+\cos\left(\frac{\pi-x}{5}\right)}{3}$

rest. princ.

$D_{f^{-1}} = \{x \in \mathbb{R} : 0 \leq \frac{\pi - x}{5} \leq \pi\} = [-4\pi; \pi]$

$0 \leq \pi - x \leq 5\pi$

$-\pi \leq -x \leq 4\pi$

$-4\pi \leq x \leq \pi$

caracterização: $f^{-1} : [-4\pi; \pi] \to [0; \frac{2}{3}]$

$x \longmapsto \frac{1}{3} + \frac{1}{3}\cos\left(\frac{\pi - x}{5}\right)$

b) $h(x) = 3\,tg\left(2x - \frac{\pi}{4}\right)$

$\cdot\ D_h = \{x \in \mathbb{R} : -\frac{\pi}{2} < 2x - \frac{\pi}{4} < \frac{\pi}{2}\} =]-\frac{\pi}{8}; \frac{3\pi}{8}[$

$-\frac{\pi}{4} < 2x < \frac{3\pi}{4}$

$-\frac{\pi}{8} < x < \frac{3\pi}{8}$

$\cdot\ y = 3\,tg\left(2x - \frac{\pi}{4}\right) \Leftrightarrow arctg\left(\frac{y}{3}\right) = 2x - \frac{\pi}{4}$

$\Leftrightarrow \dfrac{\frac{\pi}{8} + arctg\left(\frac{y}{8}\right)}{2} = x$

$h^{-1}(x) = \dfrac{\frac{\pi}{4} + arctg\left(\frac{x}{8}\right)}{2}$

$\cdot\ D_{h^{-1}} = \mathbb{R} \qquad CD_{h^{-1}} = D_h =]-\frac{\pi}{8}; \frac{3\pi}{8}[$

carac: $h^{-1} : \mathbb{R} \longrightarrow]-\frac{\pi}{8}; \frac{3\pi}{8}[$

$x \longmapsto \dfrac{\frac{\pi}{8} + arctg\left(\frac{x}{8}\right)}{2}$

Continuidade

1. Seja a função real de variável real f definida por:

$$f(x) = \begin{cases} sen\,x + e^x & se\ x < 0 \\ x - 4\ln(x) & se\ x \geq 0 \end{cases}$$

a) determine o domínio de f

$D_f = \{x \in \mathbb{R} : (x > 0 \wedge x \geq 0) \vee (\mathbb{R} \wedge x < 0)\} = \mathbb{R} \setminus \{0\}$

$x > 0 \vee x < 0$

b) f é contínua em $\mathbb{R}$? consegue encontrar (em caso negativo) um prolongamento da função f de modo a que seja contínua em $\mathbb{R}$?

$\cdot$ não, porque $D_f = \mathbb{R} \setminus \{0\}$

$\rightarrow \lim\limits_{x \to 0^+} (x - 4\ln(x)) = -(-\infty) = +\infty$

logo, não é prolongável por continuidade em $\mathbb{R}$

$\cdot$ para $x < 0$: a função f é a soma de duas funções contínuas (trigonométrica e exponencial), logo é contínua

$\cdot$ para $x > 0$: a função f é contínua, pois é a subtração de uma afim e logarítmica, ambas contínuas

c) mostre que f tem pelo menos um zero em $]-\pi; -\frac{\pi}{2}[$

$\cdot$ como $f(x)$ é contínua em $x < 0$, logo é contínua em $[-\pi; -\frac{\pi}{2}]$

$\cdot\ f\left(-\frac{\pi}{2}\right) = sen\left(-\frac{\pi}{2}\right) + e^{\frac{\pi}{2}} = -1 + \frac{1}{e^{\frac{\pi}{2}}} < 0$

$f(-\pi) = sen(-\pi) + e^{-\pi} = 0 + \frac{1}{e^{\pi}} > 0$

$\cdot$ pelo corolário do teorema Bolzano como $f(-\pi) \times f\left(-\frac{\pi}{2}\right) < 0$, então $\exists c \in]-\pi; -\frac{\pi}{2}[: f(c) = 0$

d) mostre que f tem máximo e mínimo em $[e; e^2]$

- f é contínua para $x>0$, logo é contínua em $[e; e^2]$
- $[e; e^2]$ é um intervalo fechado, limitado e não vazio
- logo, pelo teorema de Weierstrass, tem um máximo e um mínimo em $[e; e^2]$

1. Calcule os limites

a) $\lim\limits_{x\to+\infty} \dfrac{(2x+3)^3\cdot(3x-2)^2}{x^5+3} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to+\infty} \dfrac{(8x^3+\dots)(9x^2+\dots)}{x^5+\dots} = \lim\limits_{x\to+\infty} \dfrac{72\cancel{x^5}}{\cancel{x^5}} = 72$

b) $\lim\limits_{x\to+\infty} \dfrac{\sqrt{2x}}{\sqrt{3x+\sqrt{4x-1}}} \overset{\frac{\infty}{\infty}}{=} \sqrt{\lim\limits_{x\to+\infty} \dfrac{\frac{2x}{x}}{\frac{3x+\sqrt{4x-1}}{x}}} = \sqrt{\lim\limits_{x\to+\infty} \dfrac{2}{3+\underbrace{\sqrt{\frac{4x-1}{x^2}}}}} = \sqrt{\dfrac{2}{3}}$

c.aux. $\sqrt{\dfrac{4}{x}-\dfrac{1}{x^2}} \to 0$

c) $\lim\limits_{x\to-1} \dfrac{x^2+2x+1}{x^2-x-2} \overset{\frac{0}{0}}{=} \lim\limits_{x\to-1} \dfrac{(x+1)^2}{(x+1)(x-2)} = \dfrac{0}{-3} = 0$

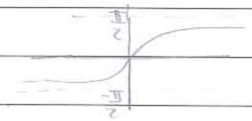
c.aux. (Ruffini)

	1	-1	-2
-1	↓	-1	2
	1	-2	0

d) $\lim\limits_{x\to0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x} \overset{\frac{0}{0}}{=} \lim\limits_{x\to0} \dfrac{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}{x(\sqrt{1+x}+\sqrt{1-x})} = \lim\limits_{x\to0} \dfrac{\cancel{1}+x-\cancel{1}+x}{x(\sqrt{1+x}+\sqrt{1-x})} =$

$= \lim\limits_{x\to0} \dfrac{2\cancel{x}}{\cancel{x}(\sqrt{1+x}+\sqrt{1-x})} = 2\lim\limits_{x\to0} \dfrac{1}{\sqrt{1+x}+\sqrt{1-x}} = \cancel{2}\times\dfrac{1}{\cancel{2}} = 1.$

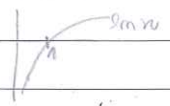
e) $\lim\limits_{x\to+\infty} \operatorname{arctg} x = \dfrac{\pi}{2}$



f) $\lim\limits_{x\to+\infty} [\ln(x^2+x) - \ln(x^2-5x+3)] \overset{\infty-\infty}{=} \ln\left|\dfrac{(x^2+x)}{(x^2-5x+3)}\right| = \ln\left[\lim\limits_{x\to+\infty} \dfrac{x^2+x}{x^2-5x+3}\right] =$

podem trocar

$= \ln\left[\lim\limits_{x\to+\infty} \dfrac{x^2}{x^2}\right] = \ln(1) = 0$



g) $\lim\limits_{x\to0} \dfrac{x^2}{\operatorname{sen}^2(3x)} \overset{\frac{0}{0}}{=} \dfrac{1}{3}\underbrace{\lim\limits_{x\to0} \dfrac{3x}{\operatorname{sen}(3x)}}_{=1} \cdot \dfrac{1}{3}\underbrace{\lim\limits_{x\to0} \dfrac{3x}{\operatorname{sen}(3x)}}_{\text{c.aux. } =1} = \dfrac{1}{9}$

$\boxed{\lim\limits_{x\to0} \dfrac{\operatorname{sen} x}{x} = 1}$ limite notável

h) $\lim\limits_{x\to+\infty} \dfrac{\operatorname{sen} x}{2x} = 0$

$-1 \leq \operatorname{sen} x \leq 1$

$-\dfrac{1}{2x} \leq \dfrac{\operatorname{sen} x}{2x} \leq \dfrac{1}{2x}$

$\lim\limits_{x\to+\infty}\left(\dfrac{-1}{2x}\right) \leq \lim\limits_{x\to+\infty}\left(\dfrac{\operatorname{sen} x}{2x}\right) \leq \lim\limits_{x\to+\infty}\left(\dfrac{1}{2x}\right)$

$0 \leq \dots\dots \leq 0$

teorema do encaixe

i) $\lim\limits_{x\to 0} \dfrac{e^{x}-e^{3x}}{3x} \overset{\frac{0}{0}}{=} -\lim\limits_{x\to 0} \dfrac{e^{x}(1+e^{2x})}{3x} = -\dfrac{1}{3}.2 \underbrace{\lim\limits_{x\to 0} e^{x}}_{=1} . \underbrace{\lim\limits_{x\to 0} \dfrac{e^{2x}-1}{2x}}_{=1} = -\dfrac{2}{3}$

j) $\lim\limits_{x\to 0} \dfrac{\ln(x+1)^5}{3x} \overset{\frac{0}{0}}{=} \dfrac{1}{3}\lim\limits_{x\to 0} \underbrace{\dfrac{5\ln(x+1)}{x}}_{=1} = \dfrac{5}{3}$ //

1. Considere $f(x) = \begin{cases} x^2 \operatorname{sen} \dfrac{1}{x} & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}$

a) estude a continuidade em $x=0$

. $\lim\limits_{x\to 0^{\pm}} x^2.\operatorname{sen}\dfrac{1}{x} = 0$

c.aux. $\to -1 \leq \operatorname{sen}\dfrac{1}{x} \leq 1 \Leftrightarrow -x^2 \leq x^2\operatorname{sen}\left(\dfrac{1}{x}\right) \leq x^2 \Leftrightarrow \underbrace{\lim\limits_{x\to 0^{\pm}} (-x^2)}_{0} \leq \lim\limits_{x\to 0^{\pm}} x^2.\operatorname{sen}\left(\dfrac{1}{x}\right) \leq \underbrace{\lim\limits_{x\to 0^{\pm}} x^2}_{0}$

. $\lim\limits_{x\to 0} f(x) = f(0) = 0$, logo $f(x)$ é contínua em $x=0$

b) estude a derivabilidade em $x=0$

. $f'(0^{\pm}) = \lim\limits_{x\to 0^{\pm}} \dfrac{x^2 \operatorname{sen}\frac{1}{x} - 0}{x-0} = \lim\limits_{x\to 0^{\pm}} x.\operatorname{sen}\dfrac{1}{x} = 0$

função limitada entre -1 e 1 multiplicada por infinitésimo é sempre 0

. $f'(0^{+}) = f'(0^{-}) = 0$ //

2. Considere $g(x) = \begin{cases} \dfrac{3-x^2}{2} & \text{se } 0 \leq x \leq 1 \\ \dfrac{\ln x}{e^{x}-1} & \text{se } 1 \leq x \leq 2 \end{cases}$

a) estude a continuidade em $x=1$

. $\lim\limits_{x\to 1^{+}} \dfrac{\ln x}{e^{x}-1} = \dfrac{0}{e-1} = 0$

$\neq$ logo $\nexists \lim\limits_{x\to 1} g(x)$; não é contínua em $x=1$

. $\lim\limits_{x\to 1^{-}} \dfrac{3-x^2}{2} = \dfrac{2}{2} = 1$

b) a função é diferenciável em $x=1$?

como a função não é contínua em $x=1$, logo não é diferenciável nesse ponto

3. Determine a equação da reta tangente à curva $y = \operatorname{sen} x$ no ponto $x=\pi$

$f(x) = \operatorname{sen} x$

$f(\pi) = \operatorname{sen}\pi = 0$ $\qquad y - f(\pi) = f'(\pi).(x-\pi)$

$f'(x) = \cos x$ $\qquad y - 0 = -1(x-\pi)$

$f'(\pi) = \cos\pi = -1$ $\qquad \boxed{y = -x+\pi}$

4. Calcule as derivadas simplificadas

a) $y = 3\sqrt{x} - \dfrac{2}{\sqrt[3]{x}} + \dfrac{7}{x}$ $\qquad y = 3x^{1/2} - 2x^{-1/3} + \dfrac{7}{x} \Leftrightarrow y' = 3.\dfrac{1}{2}x^{-1/2} - 2\left(-\dfrac{1}{3}\right)x^{-4/3} - \dfrac{7}{x^2}$

$\Leftrightarrow y' = \dfrac{3}{2\sqrt{x}} + \dfrac{2}{3\sqrt[3]{x^4}} - \dfrac{7}{x^2}$

b) $y = \frac{ax^2+b}{\sqrt{x+1}}$, $a, b \in \mathbb{R}$ $\quad y' = \frac{2ax\sqrt{x+1} - (ax^2+b)(\sqrt{x+1})'}{(\sqrt{x+1})^2} \Leftrightarrow y' = \frac{\frac{2ax\sqrt{x+1} \times 2\sqrt{x+1}}{2\sqrt{x+1}} - \frac{ax^2+b}{2\sqrt{x+1}}}{x+1}$

$\Leftrightarrow y' = \frac{\frac{4ax(x+1) - ax^2 - b}{2\sqrt{x+1}}}{x+1} = 3ax^2 + 4ax - b$

$\boxed{\sqrt[n]{u^m} = u^{m/n}}$

c) $y = tg^3 x - 3tg\,x + 3x \rightarrow$ mostrar que $y' = 3tg^4 x$

$y' = 3tg^2 x \cdot sec^2 x - 3sec^2 x + 3 \Leftrightarrow y' = 3tg^2 x(tg^2 x + 1) - 3(tg^2 x + 1) + 3$

$= 3tg^4 x + \cancel{3tg^2 x} - \cancel{3tg^2 x} - \cancel{3} + \cancel{3}$

$= 3tg^4 x$

$\boxed{sec\,x = \frac{1}{cos\,x} \qquad cosec\,x = \frac{1}{sen\,x}}$

d) $y = ln\sqrt{\frac{1+2x}{1-2x}} = \frac{1}{2} ln\left(\frac{1+2x}{1-2x}\right) = \frac{1}{2}\left[ln(1+2x) - ln(1-2x)\right]$

$y' = \frac{1}{2}\left(\frac{2}{1+2x} - \frac{-2}{1-2x}\right) = \frac{1}{\underset{(1-2x)}{1+2x}} + \frac{1}{\underset{(1+2x)}{1-2x}} = \frac{1-2x+1+2x}{1-4x^2} = \frac{2}{1-4x^2}$

Notas! $\boxed{sen^2 x + cos^2 x = 1} \xrightarrow{:sen^2 x} \boxed{1 + cotg^2 x = cosec^2 x}$

$\xrightarrow{:cos^2 x} \boxed{tg^2 x + 1 = sec^2 x}$

Resumo: regras de derivação novas

- $(arctg\,u)' = \frac{u'}{1+u^2} \longrightarrow$ • $(arccotg\,u)' = -\frac{u'}{1+u^2}$
- $(arcsen\,u)' = \frac{u'}{\sqrt{1-u^2}} \longrightarrow$ • $(arccos\,u)' = -\frac{u'}{\sqrt{1-u^2}}$
- $(u^m)' = m \cdot u^{m-1} \cdot u'$
- $(a^u)' = u' \cdot a^u \cdot ln\,a$

$\boxed{(u^v)' = v \cdot u^{v-1} \cdot u' + v' \cdot u^v \cdot ln\,u}$ u e v são funções, m e a são constantes

e) $y = (1+x^2)\,arctg\,x \qquad y' = 2x \cdot arctg\,x + (1+x^2) \cdot \frac{x'}{1+x^2} = 2x\,arctg(x) + 1$

f) $y = \frac{arcsen\,x}{\sqrt{1-x^2}} \qquad y' = \frac{\frac{1}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2} + \frac{arcsen(x) \cdot x}{\sqrt{1-x^2}}}{(\sqrt{1-x^2})^2} = \frac{\frac{\sqrt{1-x^2} + x \cdot arcsen\,x}{\sqrt{1-x^2}}}{1-x^2} =$

$= \frac{\sqrt{1-x^2} + x \cdot arcsen\,x}{\sqrt{(1-x^2)^3}}$

$\left[(1-x^2)^{\frac{1}{2}}\right]' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}}$

g) $y = x \cdot arcsen^2 x - 2x + 2\sqrt{1-x^2}\,arcsen\,x$

$y' = (x \cdot arcsen^2 x)' - (2x)' + (2\sqrt{1-x^2}\,arcsen\,x)' = arcsen^2 x + x\overbrace{(arcsen^2 x)'}^{2 \cdot arcsen\,x \cdot (arcsen\,x)'} - 2 +$

$+ 2(\sqrt{1-x^2})' \cdot arcsen\,x + (arcsen\,x)' \cdot 2\sqrt{1-x^2}$

cálculos: $(arcsen\,x)' = \frac{1}{\sqrt{1-x^2}}$

$(\sqrt{1-x^2})' = \frac{-2x}{2\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}}$

$= arcsen^2 x + \frac{2x\,arcsen\,x}{\sqrt{1-x^2}} - 2 - \frac{2x \cdot arcsen\,x}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} =$

$= arcsen^2 x - 2 + 2 = \boxed{arcsen^2 x}$

b) $y = \left(\frac{x+1}{x-1}\right)^x$ $\quad y' = x \cdot \left(\frac{x+1}{x-1}\right)^{x-1} \cdot \left(\frac{x+1}{x-1}\right)' + 1 \cdot \left(\frac{x+1}{x-1}\right)^x \cdot \ln\left(\frac{x+1}{x-1}\right)$

$$= x \cdot \frac{\left(\frac{x+1}{x-1}\right)^x \cdot \frac{x+1}{x-1}}{\cancel{\frac{x+1}{x-1}}} + \left(\frac{x+1}{x-1}\right)^x \cdot \ln\left(\frac{x+1}{x-1}\right)$$

$$= x \cdot \left(\frac{x+1}{x-1}\right)^x + \left(\frac{x+1}{x-1}\right)^x \cdot \ln\left(\frac{x+1}{x-1}\right)$$

$\frac{-2x}{x^2-1}$

2ª frequência - 11/01/2014

usar definição sempre por ramos

1. seja $g: D \subset \mathbb{R} \rightarrow \mathbb{R}$ a função definida por $g(x) = \begin{cases} e^{\frac{1}{3-x}}, & x \geq 0 \\ \operatorname{arctg}\left(\frac{x}{x+3}\right), & x < 0 \end{cases}$

a) averigue se existem assíntotas ao gráfico de g

$D_g = \{x \in \mathbb{R}: (3-x \neq 0 \wedge x \geq 0) \vee (x+3 \neq 0 \wedge x < 0\} = \mathbb{R} \setminus \{-3; 3\}$ $\quad$ A.V. $x=3$? $x=-3$? $x=0$?

$(x \neq 3 \wedge x \geq 0) \vee (x \neq -3 \wedge x < 0)$

$\mathbb{R}^+_0 \setminus \{3\} \cup \mathbb{R}^- \setminus \{3\}$

A.V.

- $\lim_{x \to 3^+} e^{\frac{1}{3-x}} = e^{\frac{1}{0^-}} = e^{-\infty} = 0$ $\quad$ • $\lim_{x \to 3^-} e^{\frac{1}{3-x}} = e^{\frac{1}{0^+}} = e^{+\infty} = +\infty \rightarrow x=3$ A.V.
- $\lim_{x \to 0^+} e^{\frac{1}{3-x}} = \sqrt[3]{e}$ $\quad$ • $\lim_{x \to 0^-} \operatorname{arctg}\left(\frac{x}{x+3}\right) = 0 \rightarrow$ não é contínua em $x=0$
- $\lim_{x \to -3^+} \operatorname{arctg}\left(\frac{x}{x+3}\right) = \operatorname{arctg}\left(-\frac{3}{0^+}\right) = \operatorname{arctg}(-\infty) = -\frac{\pi}{2}$ $\quad$ • $\lim_{x \to -3^-} \operatorname{arctg}\left(\frac{-3}{0^-}\right) = \operatorname{arctg}(+\infty) = \frac{\pi}{2}$

$\nexists$ tem ass. V em $x=-3$

b) monotonia e extremos

• $x=0$: não é diferenciável, pois não é contínua nesse ponto

$g'(x) = \begin{cases} \frac{(-3+x)'}{(3-x)^2} e^{\frac{1}{3-x}} = \frac{1 \cdot e^{\frac{1}{3-x}}}{(3-x)^2} \\ \frac{\left(\frac{x}{x+3}\right)'}{1+\left(\frac{x}{x+3}\right)^2} = \frac{\frac{x+3-x}{(x+3)^2}}{(x+3)^2 + \frac{x^2}{(x+3)^2}} = \frac{3}{x^2+6x+9+x^2} = \frac{3}{2x^2+6x+9} \end{cases}$

$g'(x) = 0 \Leftrightarrow (e^{\frac{1}{3-x}} = 0 \wedge (3-x^2 \neq 0) \wedge x \geq 0 \rightarrow$ imp.

$g'(x) = 0 \Leftrightarrow 3 = 0 \wedge 2x^2+6x+9 \neq 0 \wedge x < 0$ imp.

c) sentido das concavidades e pontos de inflexão

$g''(x) = \begin{cases} \frac{7-2x}{(3-x)^4} \cdot e^{\frac{1}{3-x}}, & x \geq 0 \\ \frac{-6(2x+3)}{(2x^2+6x+9)^2}, & x \leq 0 \end{cases}$

$g''(x) = 0 \Leftrightarrow \left(x = \frac{7}{2} \wedge x \geq 0\right) \vee \left(x = -\frac{3}{2} \wedge x < 0\right)$

	$-\infty$	-3		$-3/2$		0		3		$7/2$	$+\infty$
$g''(x)$	+	X	+	0	−	X	+	X	+	0	−
$g(x)$	∪	X	∪	PI	∩	PI	∪	X	∪	PI	∩

1. A.N.V = $m = \lim_{x \to +\infty} \frac{e^{\frac{1}{3-x}}}{x} = \frac{1}{+\infty} = 0$ | $b = e^{\frac{1}{3-x}} = 1$ $\quad y=1 \rightarrow$ ASS. h.

$m = \lim_{x \to -\infty} \frac{\operatorname{arctg}\left(\frac{x}{x+3}\right)}{x} = \frac{\frac{\pi}{4}}{-\infty} = 0$ | $b = \lim_{x \to -\infty} \operatorname{arctg}\left(\frac{x}{x+3}\right) = \frac{\pi}{4} \rightarrow$ As. h $y = \frac{\pi}{4}$

2. g'_2 $\quad g'_1$

	$-\infty$	-3		0		3	$+\infty$
$g'(x)$	+	X	+	X	+	X	+
$g(x)$	↗	X	↗	$\sqrt[3]{e}$	↗	X	↗

- g é crescente em todo o seu domínio
- não tem extremos

1. Calcule

a) $\lim\limits_{x\to 0^+} (\operatorname{sen}x+\cos x)^{\ln x} \overset{1^\infty}{=} A$

$$\ln A = \lim_{x\to 0^+} \ln x \cdot \ln(\operatorname{sen}x+\cos x) \overset{\infty\times 0}{\Leftrightarrow} \ln A = \lim_{x\to 0^+} \frac{\ln(\operatorname{sen}x+\cos x)}{\left(\frac{1}{\ln x}\right)} \underset{R.C.}{\overset{\frac{0}{0}}{=}}$$

$$= -\lim_{x\to 0^+} \frac{\frac{\cos x-\operatorname{sen}x}{\operatorname{sen}x+\cos x}}{\frac{1}{x\ln^2 x}} = -\lim_{x\to 0^+}\left[\underbrace{\frac{\cos x-\operatorname{sen}x}{\operatorname{sen}x+\cos x}}_{\to 1}\cdot \underbrace{x\ln^2 x}_{0\times\infty}\right] = -\lim_{x\to 0^+}\frac{\ln^2 x}{\left(\frac{1}{x}\right)} \underset{R.C.}{\overset{\frac{\infty}{\infty}}{=}}$$

C. aux. $\cdot$ $\dfrac{\cos x-\operatorname{sen}x}{\operatorname{sen}x+\cos x}$

$\cdot \left(\dfrac{1}{\ln x}\right)' = [(\ln x)^{-1}]' = -(\ln x)^{-2}\cdot\dfrac{1}{x} = -\dfrac{1}{x\ln^2 x}$

$$= -\lim_{x\to 0^+}\frac{2\ln x\cdot\frac{1}{x}}{-\frac{1}{x^2}} = +2\lim_{x\to 0^+}\underbrace{\frac{\ln x}{x}\cdot x^2}_{\infty\times 0} =$$

$$= 2\lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}} \underset{R.C}{\overset{\frac{\infty}{\infty}}{=}} 2\lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}} =$$

$$= -2\lim_{x\to 0^+}\frac{1}{x}\times x^2 = -2(0)=0$$

$\ln A = 0 \Leftrightarrow A = e^0 = 1$ //

b) $\lim\limits_{x\to +\infty} (1+\ln x)^{\frac{1}{\sqrt{\ln x}}} \overset{\infty^0}{=} A$

$$\ln A = \lim_{x\to+\infty}\frac{1}{\sqrt{\ln x}}\cdot\ln(1+\ln x) \overset{0\times\infty}{=} \lim_{x\to+\infty}\frac{\ln(1+\ln x)}{\sqrt{\ln x}} \underset{R.C.}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to+\infty}\frac{\frac{1}{x(1+\ln x)}}{\frac{1}{2x\sqrt{\ln x}}} =$$

$$=2\lim_{x\to+\infty}\frac{\sqrt{\ln x}}{1+\ln x} \underset{R.C.}{\overset{\frac{\infty}{\infty}}{=}} 2\lim_{x\to+\infty}\frac{\frac{1}{2x\sqrt{\ln x}}}{\frac{1}{x}} = \cancel{2}\lim_{x\to+\infty}\frac{1}{\cancel{2}x\sqrt{\ln x}}\cdot x = 0 \text{ logo}$$

$\ln A = 0 \Leftrightarrow A = e^0 = 1$ //

c. aux. $\cdot$ $[\ln(1+\ln x)]' = \dfrac{0+\frac{1}{x}}{1+\ln x} = \dfrac{\frac{1}{x}}{1+\ln x} = \dfrac{1}{x(1+\ln x)}$

$\cdot \left[(\ln x)^{\frac{1}{2}}\right]' = \dfrac{1}{2}(\ln x)^{-\frac{1}{2}}\cdot\dfrac{1}{x} = \dfrac{1}{2}\cdot\dfrac{1}{(\ln x)^{1/2}}\cdot\dfrac{1}{x} = \dfrac{1}{2x\sqrt{\ln x}}$

c) $\lim\limits_{x\to 0^+} (1+e^x-2\cos^2 x)^{\operatorname{sen}x} \overset{0^0}{=} A$

$$\ln A = \lim_{x\to 0^+}\operatorname{sen}x\cdot\ln(1+e^x-2\cos^2 x) \overset{0\times\infty}{=} \lim_{x\to 0^+}\frac{(\ln(1+e^x-2\cos^2 x))'}{\left(\frac{1}{\operatorname{sen}x}\right)'} \underset{R.C.}{\overset{\frac{\infty}{\infty}}{=}}$$

$$= \frac{\frac{e^x+4\cos x\cdot\operatorname{sen}x}{1+e^x-2\cos^2 x}}{-\operatorname{cosec}x\cdot\operatorname{cotg}x \longrightarrow -\frac{\cos x}{\operatorname{sen}^2 x}} = -\lim_{x\to 0^+}\frac{\overbrace{e^x+4\cos x\cdot\operatorname{sen}x}^{=1}}{\underbrace{1+e^x-2\cos^2 x}_{\to 0}}\cdot\frac{\overbrace{\operatorname{sen}^2 x}^{\to 0}}{\underbrace{\cos x}_{\to 1}} =$$

$$= -\lim_{x\to 0^+}\frac{e^x+4\cos x\cdot\operatorname{sen}x}{\cos x}\cdot\lim_{x\to 0^+}\frac{\operatorname{sen}^2 x}{1+e^x-2\cos^2 x} \underset{R.C.}{\overset{\frac{0}{0}}{=}} -1\times\lim_{x\to 0^+}\frac{2\operatorname{sen}x\cdot\cos x}{e^x+4\cos x\cdot\operatorname{sen}x} = -1\times 0 = 0$$

como $\ln A = 0 \Leftrightarrow A = e^0 = 1$ //

by Staples

2º Mini-teste - 24/11/2012 - Grupo II - ex 2

a) determine $f'(0)$, sabendo que $f(x) = g(\text{sen}^2 x) - 2g^3(e^{x^3} x^2)$ e que é diferenciável em $\mathbb{R}$

$$f'(x) = g'(\text{sen}^2 x).(\text{sen}^2 x)' - 2\times 3[g(e^{x^3}.x^2)]^2 \times (g(e^{x^3}.x^2))' =$$

$$= g'(\text{sen}^2 x).2\,\text{sen}x.\cos x - 6[g(e^{x^3}.x^2)]^2.g'(e^{x^3}.x^2).(e^{x^3}.x^2)' =$$

$$= g'(\text{sen}^2 x).2\,\text{sen}(2x) - 6[g(e^{x^3}x^2)]^2.g'(e^{x^3}.x^2).(3x^2.e^{x^3}.x^2 + e^{x^3}.2x)$$

b) calcule $(f^{-1})'(\frac{5\pi}{4})$ para $f(x) = 5\,\text{arctg}(\frac{x}{3})$

$\times\ f'(0) = \underbrace{g'(0).\text{sen}(0)}_{=0} - \underbrace{6g^2(0).g'(0).2.0.e^0}_{=0} = 0$

→ $f(x) = 5\,\text{arctg}(x/3) = \frac{5\pi}{4} \Leftrightarrow \frac{x}{3} = \text{tg}\left(\frac{\pi}{4}\right) \Leftrightarrow x = 3$

. $\frac{1}{f'(x)} = \frac{1}{5\,\text{arctg}(\frac{x}{3})'} = \frac{1}{5\times\frac{(\frac{x}{3})'}{1+(\frac{x}{3})^2}} = \frac{1}{5\times\frac{\frac13}{\frac{9+x^2}{9}}} = \frac{9+x^2}{15}$. $\frac{1}{f'(3)} = \frac{9+3^2}{15} = \frac{6}{5}$

→ $f^{-1}(\frac{5\pi}{4}) = \frac{5}{3(9+\frac{5\pi}{4})}$

. $y = 5\,\text{arctg}(\frac{x}{3}) \Leftrightarrow 3\,\text{tg}(\frac{y}{5}) = x$

. $3\,\text{tg}(\frac{x}{5})' = 3\times\frac{(\frac{x}{5})'}{\cos^2(\frac{x}{5})} = \frac{3\times\frac15}{\cos^2(\frac{x}{5})}$. $[f^{-1}(\frac{5\pi}{4})]' = 3\times\frac{1}{5.\cos^2\left(\frac{\frac{5\pi}{4}}{5}\right)}$

$= \frac35\times\frac{1}{\frac12} = \frac65$

1. Determinar $a, b \in \mathbb{R}$ tais que:

1º ver indeterminação ($\frac00$; $\frac{\infty}{\infty}$)
2º pôr constantes em evidência
3º pôr sen x/cos x em evidência

a) $\lim_{x\to0}\frac{x(1+a\cos x) - b\,\text{sen}x}{x^3} = 1$

$\frac00$ $\stackrel{R.C.}{=} \lim_{x\to0}\frac{1+a\cos x + x(-a\,\text{sen}x) - b.\cos x}{3x^2} = \frac13\lim_{x\to0}\frac{1+\overbrace{(a-b)\cos x}^{a-b} - \overbrace{ax.\text{sen}x}^{0}}{x^2 \to 0}$

$\frac{1+a-b}{0}$ — $\frac{\infty}{\infty}\to\infty$ ✗ — $\frac{K}{0}\to\infty\ (K\neq0)$ ✗ — $\frac00$ ✓

se $\boxed{1+a-b=0}$ então

$= \frac13\lim_{x\to0}\frac{(-a+b)\,\text{sen}x - a\,\text{sen}x - ax\cos x}{2x} = \frac16\lim_{x\to0}\frac{(b-2a)\,\text{sen}x - ax\cos x}{x}$

$\frac00 \stackrel{R.C.}{=} \frac16\lim_{x\to0}\frac{(b-2a)\cos x - a\cos x + ax\,\text{sen}x}{1} = \frac16\lim_{x\to0}[(b-3a)\cos x + ax\,\text{sen}x]$

$= \frac{b-3a}{6}$. $\begin{cases}1+a-b=0\\ \frac{b-3a}{6}=1\end{cases} \Leftrightarrow \begin{cases} b = -\frac32 \\ a = -\frac52\end{cases}$

b) $\lim_{x\to0}(e^{ax} - bx)^{\frac{1}{x^2}} = \sqrt{e}$ $\ln\sqrt{e} = \ln e^{\frac12} = \frac12\times1 = \frac12$

1^∞

$= \lim_{x\to0}\frac{1}{x^2}.\ln(e^{ax}-bx) = \lim_{x\to0}\frac{\ln(e^{ax}-bx)}{x^2} \stackrel{\frac00}{\underset{R.C.}{=}} \lim_{x\to0}\frac{\frac{ae^{ax}-b}{e^{ax}-bx}}{2x} = \lim_{x\to0}\frac{ae^{ax}-b}{2x(e^{ax}-bx)}$

$\frac{a-b}{0}$!

se $a-b=0$ então $\lim_{x\to0}\frac{a^2e^{ax}}{2(e^{ax}-bx)+2x(ae^{ax}-b)} = \frac{a^2}{2}$ $\begin{cases}\frac{a^2}{2}=\frac12\\a-b=0\end{cases}\Leftrightarrow\begin{cases}a=\pm1\\b=\pm1\end{cases}$

Primitivas

1 Calcule

a) $P\left(x - \frac{x^2}{2} + \frac{1}{x^3}\right) = Px - \frac{1}{2}Px^2 + Px^{-3} = \frac{x^2}{2} - \frac{1}{2}\frac{x^3}{3} + \frac{x^{-2}}{-2} + C$

regra da potência

$= \frac{x^2}{2} - \frac{x^3}{6} - \frac{1}{2x^2} + C, C \in \mathbb{R}$

b) $P\left(\frac{1+\sqrt[3]{x}}{\sqrt{x}}\right)^2 = P\frac{1+2\sqrt[3]{x}+\sqrt[3]{x^2}}{x} = P\frac{1}{x} + 2P\frac{x^{1/3}}{x} + P\frac{x^{2/3}}{x} =$

$= P\frac{\overset{u'}{1}}{\underset{u}{x}} + 2Px^{-2/3} + Px^{-1/3} = \ln|x| + 2\frac{x^{1/3}}{1/3} + \frac{x^{2/3}}{2/3} + C$

$= \ln|x| + 2\times 3\sqrt[3]{x} + \frac{3}{2}\sqrt[3]{x^2} + C = \ln|x| + 6\sqrt[3]{x} + \frac{3}{2}\sqrt[3]{x^2} + C, C \in \mathbb{R}$

c) $Px(2x^2+3)^{1/2} = \frac{1}{4}P\underbrace{4x}_{u'}\underbrace{(2x^2+3)^{1/2}}_{u^\alpha} = \frac{1}{4}\frac{(2x^2+3)^{3/2}}{3/2} + C = \frac{1}{4}\cdot\frac{2}{3}\sqrt{(2x^2+3)^3} + C =$

$= \frac{\sqrt{(2x^3+3)^3}}{6} + C, C \in \mathbb{R}$

d) $P\,tg\,x = -P\frac{\overset{u'}{-sen\,x}}{\underset{u}{cos\,x}} = -\ln|cos\,x| + C, C \in \mathbb{R}$ $\qquad P\frac{u'}{u} = \ln|u|$

e) $P\underbrace{e^{5x}}_{e^u \cdot u'} = \frac{1}{5}Pe^{5x}\cdot 5 = \frac{1}{5}e^{5x} + C, C \in \mathbb{R}$

f) $Pe^{3x+3} = \frac{1}{3}Pe^{3x+3}\cdot 3 = \frac{1}{3}e^{3x+3} + C, C \in \mathbb{R}$

g) $P\underbrace{2x^3}_{u'}\underbrace{e^{x^4}}_{e^u} = \frac{1}{2}P\,2\cdot 2x^3\cdot e^{x^4} = \frac{1}{2}e^{x^4} + C, C \in \mathbb{R}$

h) $P\frac{e^{2x}-1}{e^x} = P\frac{e^{2x}}{e^x} - P\frac{1}{e^x} = Pe^x + P(-e^{-x}) = e^x + e^{-x} + C = e^x + \frac{1}{e^x} + C =$

$= \frac{e^{2x}+1}{e^x} + C, C \in \mathbb{R}$

i) $P\frac{e^{3x}}{2+5e^{3x}} = \frac{1}{15}P\frac{\overset{u'}{15e^{3x}}}{\underset{u}{2+5e^{3x}}} = \frac{\ln(2+5e^{3x})}{15} + C, C \in \mathbb{R}$

j) $P\frac{e^x}{e^{2x}+1} = P\frac{\overset{u'}{e^x}}{1+(\underset{u}{e^x})^2} = arctg(e^x) + C, C \in \mathbb{R}$

k) $P\frac{\overset{u'}{x^2+x}}{\underset{u}{2x^3+3x^2+1}} = \frac{1}{6}P\frac{6x^2+6x}{2x^3+3x^2+1} = \frac{1}{6}\ln|(2x^3+3x^2+1)| + C, C \in \mathbb{R}$

$6x^2+6x$

l) $P\frac{5}{1+9x^2} = 5P\frac{1}{1+(3x)^2} = \frac{5}{3}P\frac{3}{1+(3x)^2} = \frac{5}{3}arctg(3x) + C, C \in \mathbb{R}$

m) $P\, \underbrace{sen^4 x}_{u^\alpha} \cdot \underbrace{\cos x}_{u'} = \frac{sen^5 x}{5} + C,\ C \in \mathbb{R}$

n) $P\, \frac{sen\, x}{\cos^5 x} = -P \underbrace{-sen\, x}_{u'} \cdot \underbrace{\cos^{-5} x}_{u^\alpha} = -\frac{\cos^{-4} x}{-4} + C = \frac{1}{4\cos^4 x} + C,\ C \in \mathbb{R}$

o) $P\, \frac{\cos(2x)}{\sqrt{sen^3(2x)}} = P\, \frac{\cos(2x)}{(sen^3(2x))^{1/2}} = \frac{1}{2} P\, \underbrace{2\cos(2x)}_{u'} \times \underbrace{(sen(2x))^{-3/2}}_{u^\alpha} = \frac{1}{2} \frac{[sen(2x)]^{-1/2}}{-1/2} + C =$

$= -\frac{1}{\sqrt{sen(2x)}} + C,\ C \in \mathbb{R}$

p) $P\, \frac{1}{x \cdot \ln x} = P\, \frac{\overset{u'}{\frac{1}{x}}}{\underbrace{\ln x}_{u}} = \ln|\ln x| + C,\ C \in \mathbb{R}$

dos mais importantes

q) $P\, \frac{\ln \sqrt{x}}{x} = \frac{1}{2} P\, \frac{\ln x}{x} = \frac{1}{2} P\, \underbrace{\frac{1}{x}}_{u'} \cdot \underbrace{\ln x^1}_{u^\alpha} = \frac{1}{2} \frac{\ln^2 x}{2} + C = \frac{\ln^2 x}{4} + C,\ C \in \mathbb{R}$

r) $P\, \frac{3 \cdot 2^x - 2 \cdot 3^x}{2^x} = P\, \frac{3 \cdot 2^x}{2^x} - P\, \frac{2 \cdot 3^x}{2^x} = 3P\underbrace{(1)}_{1=(x)'} - 2P\left(\frac{3}{2}\right)^x = 3x - 2\frac{\left(\frac{3}{2}\right)^x}{\ln\left(\frac{3}{2}\right)} =$

$P a^u u' = \frac{a^u}{\ln a}$

$= 3x - \left(\frac{3}{2}\right)^x \cdot \frac{2}{\ln\left(\frac{3}{2}\right)} + C,\ C \in \mathbb{R}$

s) $P\, \frac{5 + 3x + arctg\, x}{1+x^2} = P\, \frac{5}{1+x^2} + P\, \frac{3x}{1+x^2} + P\, \frac{arctg\, x}{1+x^2} = 5P\, \frac{1}{1+(x)^2} + P\, \frac{3x}{1+x^2} + P\, \frac{arctg\, x}{1+x^2} =$

$= 5\,arctg(x) + \frac{3}{2} P\, \frac{\overset{u'}{2x}}{1+\underbrace{(x)}_{u}^2} = 5\,arctg(x) + \frac{3}{2}\ln|1+x^2| + \underbrace{\frac{1}{1+x^2}}_{u'} \cdot \underbrace{arctg\, x}_{u^\alpha}$

$= 5\,arctg(x) + \frac{3}{2}\ln(1+x^2) + \frac{arctg^2 x}{2} + C,\ C \in \mathbb{R}$

t) $P\, \frac{7}{9+4x^2} = 7P\, \frac{1}{9\left(1+\frac{4x^2}{9}\right)} = \frac{7}{9} \cdot \frac{3}{2} P\, \frac{\overset{u'}{1 \cdot \frac{2}{3}}}{1+\underbrace{\left(\frac{2x}{3}\right)}_{u}^2} = \frac{7}{6} arctg\left(\frac{2x}{3}\right) + C,\ C \in \mathbb{R}$

$P\, \frac{u'}{1+u^2} = arctg(u)$

ou

$\boxed{P\, \frac{u'}{a^2+u^2} = \frac{1}{a} arctg\left(\frac{u}{a}\right)}$ então $P\, \frac{\overset{u'}{1 \times 2}}{\underbrace{3}_{a}^2 + \underbrace{(2x)}_{u}^2} = \frac{7}{2} \cdot \frac{1}{3} arctg\left(\frac{2x}{3}\right) + C = \frac{7}{6} arctg\left(\frac{2x}{3}\right) + C$

u) $P\, \frac{1}{\sqrt{4-25x^2}}$

$= P\, \frac{1}{\sqrt{4-25x^2}} = P\, \frac{1}{\sqrt{4\left(1-\frac{25x^2}{4}\right)}} = \frac{1}{2} \cdot \frac{2}{5} \frac{\overset{u'}{1 \cdot \frac{5}{2}}}{\sqrt{1-\underbrace{\left(\frac{5}{2}x\right)}_{u}^2}}$

ou

sabendo que $P\, \frac{u'}{\sqrt{a^2-u^2}} = arcsen\left(\frac{u}{a}\right)$ $\quad \frac{1}{5} P\, \frac{5}{\sqrt{\underbrace{2}_{a}^2 - \underbrace{(5x)}_{u}^2}} = \frac{1}{5} arcsen\left(\frac{5x}{2}\right) + C,\ C \in \mathbb{R}$

2ª Frequência 11/01/2008 - C II - ex 4

a) $P\, \sqrt{\sec x} \cdot tg\, x = P\left(\frac{1}{\cos x}\right)^{\frac{1}{2}} \cdot tg\, x = P\, \cos x^{-\frac{1}{2}} \cdot \frac{sen\, x}{\cos x} = -P\, \cos^{-\frac{3}{2}} x \cdot (-sen\, x)$

$= \frac{\cos^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C = \frac{\cos^{-\frac{1}{2}}}{+\frac{1}{2}} + C = \frac{2}{\sqrt{\cos x}} + C,\ C \in \mathbb{R}$

b) $P\,\frac{x^3+x\cdot\operatorname{arctg}(x^2)}{1+x^4} = P\,\frac{x^3}{1+x^4} + P\,\frac{x\cdot\operatorname{arctg}(x^2)}{1+x^4} = \frac{1}{4}P\,\frac{4x^3}{1+x^4} + P\left(\frac{x}{1+x^4}\cdot\operatorname{arctg}(x^2)\right)$

$= \frac{\ln(1+x^4)}{4} + P\,\frac{(x^2)'}{1+(x^2)^2}\cdot\operatorname{arctg}(x^2) = \frac{\ln(1+x^4)}{4} + \frac{1}{2}P\left(\frac{2x}{1+x^4}\cdot\operatorname{arctg}(x^2)\right) =$

$= \frac{\ln(1+x^4)}{4} + \frac{1}{2}\frac{\operatorname{arctg}^2 x^2}{2} + C,\ C\in\mathbb{R}$

c) $P\,\frac{\sqrt{3+\frac{5}{e^{2x}}}}{e^{2x}}$

$= P\,(3+5e^{-2x})^{\frac{1}{2}}\cdot e^{-2x} = -\frac{1}{10}P\,(3+5e^{-2x})^{1/2}\cdot e^{-2x}(-10) = -\frac{1}{10}\frac{(3+5e^{-2x})^{3/2}}{3/2} + C$

c.aux. $-10e^{-2x}$

$= -\frac{1}{10}\cdot\frac{2}{3}\sqrt{\left(3+\frac{5}{e^{2x}}\right)^3} + C = -\frac{1}{15}\sqrt{\left(3+\frac{5}{e^{2x}}\right)^3} + C,\ C\in\mathbb{R}$

2ª Frequência 11/01/2014

i) $P\,\frac{x+3}{\sqrt{1-4x^2}} = P\,\frac{x+3}{\sqrt{1-(2x)^2}} = P\,\frac{x}{\sqrt{1-(4x^2)}} + \frac{3}{2}P\,\frac{1\times 2}{\sqrt{1-(2x)^2}} = -\frac{1}{8}P\,(-8)x\,(1-4x^2)^{-1/2} + \frac{3}{2}\operatorname{arcsen}(2x)$

$= \frac{1}{8}\frac{(1-4x^2)^{1/2}}{1/2} + \frac{3}{2}\operatorname{arcsen}(2x) + C = -\frac{\sqrt{1-4x^2}}{4} + \frac{3}{2}\operatorname{arcsen}(2x) + C,\ C\in\mathbb{R}$

ii) $P\,\frac{e^{\operatorname{arctg}(2x)} + x\cdot\ln(1+4x^2)+1}{1+4x^2} = P\,\frac{e^{\operatorname{arctg}(2x)}}{1+4x^2} + P\,\frac{x\cdot\ln(1+4x^2)}{1+4x^2} + P\,\frac{1}{1+4x^2} =$

$= \frac{1}{2}P\,\frac{1\times 2}{1+4x^2}\cdot e^{\operatorname{arctg}(2x)} + P\,\frac{x}{1+4x^2}\cdot\ln(1+4x^2) + P\,\frac{1\times 2}{1+(2x)^2} =$

$= \frac{1}{2}e^{\operatorname{arctg}(2x)} + \frac{1}{8}\frac{\ln^2(1+4x^2)}{2} + \frac{1}{2}\operatorname{arctg}(2x) + C = \frac{1}{2}\frac{e^{\operatorname{arctg}(2x)}}{2} + \frac{\ln^2(1+4x^2)}{16} + \frac{\operatorname{arctg}(2x)}{2} + C,\ C\in\mathbb{R}$

1. Calcule

a) $P\cos^5 x = P\cos x\cdot\cos^4 x = P\cos x\,(\cos^2 x)^2 = P\cos x\,(1-\operatorname{sen}^2 x)^2 =$

$= P\cos x\,(1-2\operatorname{sen}^2 x+\operatorname{sen}^4 x) = P\,(\cos x - 2\cos x\cdot\operatorname{sen}^2 x + \cos x\cdot\operatorname{sen}^4 x)$

$= P\cos x - 2P\cos x\cdot\operatorname{sen}^2 x + P\cos x\cdot\operatorname{sen}^4 x = \operatorname{sen} x - 2\frac{\operatorname{sen}^3 x}{3} + \frac{\operatorname{sen}^5 x}{5} + C,\ C\in\mathbb{R}$

$P\operatorname{sen}^4 x = P(\operatorname{sen}^2 x)^2 = P\left(\frac{1-\cos(2x)}{2}\right)^2 = P\,\frac{1-2\cos(2x)+\cos^2(2x)}{4} =$

$= \frac{1}{4}\left[P(1) - P\,2\cos(2x) + P\cos^2(2x)\right] = \frac{x}{4} - \frac{\operatorname{sen}(2x)}{4} + \frac{1}{4}P\,\frac{1+\cos(4x)}{2} =$

$= \ldots \frac{1}{8}\left[P(1) + \frac{1}{4}P\,4\cos(4x)\right] = \ldots \frac{x}{4} - \frac{\operatorname{sen}(2x)}{4} + \frac{x}{8} + \frac{1}{32}\operatorname{sen}(4x) = \frac{3x}{8} - \frac{\operatorname{sen}(2x)}{4} + \frac{\operatorname{sen}(4x)}{32}$

$P\operatorname{sen}^2 x\cdot\cos^3 x$

$\hookrightarrow = P\operatorname{sen}^2 x\cdot\cos x\cdot\cos^2 x = P\operatorname{sen}^2 x\cdot\cos x\,(1-\operatorname{sen}^2 x) = P\operatorname{sen}^2 x\cos x - P\operatorname{sen}^4 x\cdot\cos x =$

$= \frac{\operatorname{sen}^3 x}{3} - \frac{\operatorname{sen}^5 x}{5} + C,\ C\in\mathbb{R}$

2ª Frequência 11/01/2014

$\int \frac{\operatorname{tg}^2 x}{\sec^7 x}\,dx \quad P\,\frac{\frac{\operatorname{sen}^2 x}{\cos^2 x}}{\frac{1}{\cos^7 x}} = P\,\frac{\operatorname{sen}^2 x}{\cos^2 x}\cdot\cos^7 x = P\operatorname{sen}^2 x\cdot\cos^5 x = P\operatorname{sen}^2 x\cdot\cos x\cdot\cos^4 x =$

$= P\operatorname{sen}^2 x\cdot\cos x\,(\cos^2 x)^2 = P\operatorname{sen}^2 x\cdot\cos x\,(1-\operatorname{sen}^2 x)^2 = P\operatorname{sen}^2 x\cdot\cos x\,(1-2\operatorname{sen}^2 x + \operatorname{sen}^4 x)$

$= P\,sen^2x.\cos x - 2P\,sen^4x\cos x + P\,sen^6x\cos x = \frac{sen^3x}{3} - 2\frac{sen^5x}{5} + \frac{sen^7x}{7} + C,\ C \in \mathbb{R}$

1. Calcule

a) $\int_0^4 \sqrt{2x+1}\,dx = \frac{1}{2}\int_0^4 \underbrace{2}_{u'}\underbrace{(2x+1)}_{u^\alpha}^{1/2}dx = \frac{1}{2}\left[\frac{(2x+1)^{3/2}}{3/2}\right]_0^4 = \frac{1}{2}\cdot\frac{2}{3}\left[\sqrt{(2x+1)^3}\right]_0^4 =$

$= \frac{1}{3}(\sqrt{9^3}-1) = \frac{1}{3}(\sqrt{9^2}.\sqrt{9}) - \frac{1}{3} = 9 - \frac{1}{3} = \frac{26}{3}$

b) $\int_4^9 \frac{\sqrt{z}-1}{z}\,dz \overset{c.aux.}{=} \left[2\sqrt{z} - \ln|z|\right]_4^9 =$

$= 2\sqrt{9} - \ln 9 - 2\sqrt{4} + \ln 4$

c.aux $P\frac{\sqrt{z}}{z} - P\frac{1}{z} =$

$= Pz^{-1/2} - \ln|z| = \frac{z^{1/2}}{1/2} - \ldots$

$= 2\sqrt{z} - \ln|z| + C$

c) $\int_0^{\frac{\pi}{4}} sen^3(2x)\cos(2x)\,dx$

$= \frac{1}{8}\left[sen^4(2x)\right]_0^{\frac{\pi}{4}} =$

$= \frac{1}{8}\left(sen^4\frac{\pi}{2} - 0\right) = \frac{1}{8}$

c.aux.

$\frac{1}{2}P\,\underbrace{sen^3(2x)}_{u^\alpha}\,\underbrace{\cos(2x).2}_{u'} = \frac{1}{2}\frac{sen^4(2x)}{4} + C$

$= \frac{1}{8}sen^4(2x) + C$

2ª Frequência 11/01/2013

i) $\int_0^{\frac{\pi}{4}} \frac{1+2\,sen x}{\cos^2x}\,dx = 1 + 2\sqrt{2} - (2.1+0)$

$= 2\sqrt{2}+1$

c.aux $P\frac{1}{\cos^2x} + 2P\frac{sen x}{\cos^2x} =$

$P\sec^2x - 2P\,\underbrace{-sen x}_{u'}.\underbrace{\cos^{-2}x}_{u^\alpha} = tg\,x - 2\frac{\cos^{-1}x}{-1} + C$

$= tg\,x + (2.P\frac{1}{\cos^2x}) \to \sec x$

Sebenta

4.d) $\int_4^9 \frac{x-1}{\sqrt{x}+1}\,dx$

$= \frac{9^{3/2}}{3/2} - 9 - \frac{4^{3/2}}{3/2} + 4 =$

$= \frac{2\sqrt{9^3}}{3} - 9 - \frac{2\sqrt{4^3}}{3} + 4 =$

$= \frac{2(\sqrt{9}.\sqrt{9^2})}{\cancel{3}} - 9 - \frac{2(\sqrt{4^2}.\sqrt{4})}{3} + 4 = 18 - 9 - \frac{16}{3} + 4 = \frac{39-16}{3} = \frac{23}{3}$

c.aux $\frac{(x-1)(\sqrt{x}-1)}{(\sqrt{x}+1)(\sqrt{x}-1)} = \frac{\cancel{(x-1)}(\sqrt{x}-1)}{\cancel{x-1}} =$

$= P\sqrt{x} - 1 = P\underbrace{1}_{u'}\underbrace{(x)}_{u^\alpha}^{1/2} - P(1) =$

$= \frac{x^{3/2}}{3/2} - x$

j) $\int_{-2}^{1} \frac{x+|x|}{x-|x|+2}\,dx$

$|x| = \begin{cases} x & se\ x \geq 0 \\ -x & se\ x < 0 \end{cases}$

$\int_{-2}^{0}\frac{x-x}{x+x+2}dx + \int_0^1\frac{x+x}{x-x+2} = \int_{-2}^{0}0\,dx + \int_0^1\frac{2x}{2}dx = 0 + \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$

$\text{arctg} \quad \frac{u'}{1+u^2} \quad u'$

exemplo importante !!! $\displaystyle\int_{-2}^{2} \frac{x+|x-1|}{x-|x+1|+3}\,dx$

$|x-1| = \begin{cases} x-1 & \text{se } x \geq 1 \\ -x+1 & \text{se } x < 1 \end{cases}$

$|x+1| = \begin{cases} x+1 & \text{se } x \geq -1 \\ -x-1 & \text{se } x < -1 \end{cases}$

	-2 … -1	-1 …	… 1	1 …	… 2
$\|x-1\|$	$-x+1$	$-x+1$	$-x+1$	$x-1$	$x-1$
$\|x+1\|$	$-x-1$	$x+1$	$x+1$	$x+1$	$x+1$

$\displaystyle\int_{-2}^{-1} \frac{x-x+1}{x-(-x-1)+3} \quad \int\ldots \quad \int\ldots$

- $\displaystyle\int_{\arccos(\frac{1}{4})}^{\pi/3} P\, \text{sen}\,x \sqrt{\frac{1-2\sqrt{\cos x}}{\cos x}}\,dx = P\,(\text{sen}\,x)\frac{\sqrt{1-2\sqrt{\cos x}}}{\sqrt{\cos x}} =$

$= \dfrac{(1-2\sqrt{\cos x})^{3/2}}{3/2} + C = \dfrac{2}{3}\sqrt{(1-2\sqrt{\cos x})^3} + C,\ C \in \mathbb{R}$

calc. aux.

$(1-2\sqrt{\cos u})' = -2 \cdot \frac{1}{2}(\cos u)^{-1/2}(-\text{sen}\,x)$
$= +\dfrac{\text{sen}\,u}{\sqrt{\cos x}}$

$= \dfrac{2}{3}\left[\sqrt{(1-2\sqrt{\cos x})^3}\right]_{\arccos\frac{1}{4}}^{\frac{\pi}{3}} = \dfrac{2}{3}\Big[\sqrt{(1-2\sqrt{\tfrac{1}{2}})^3} - \underbrace{\sqrt{(1-2\sqrt{\tfrac{1}{4}})^3}}_{0}\Big] = \dfrac{2}{3}\sqrt{\left(1-\dfrac{2}{\sqrt{2}}\right)^3} = \dfrac{2}{3}\sqrt{(1-\sqrt{2})^3}$

1. Calcule

a) $\displaystyle\int_0^{+\infty} \frac{dx}{x^2+2x+2}$ $\quad D = \{x \in \mathbb{R} : x^2+2x+2 \neq 0\} = \mathbb{R}$ $\quad$ não há problemas no D, logo é 1ª espécie

(1ª espécie) $\quad x = \dfrac{-2 \pm \sqrt{4-4(1)(2)}}{2} \rightarrow$ não tem zeros

$= \displaystyle\lim_{b \to +\infty} \int_0^b \frac{1}{x^2+2x+2}\,dx$

metade

$\underbrace{x^2+2x+(1)^2}_{(x+1)^2} \underbrace{+2-(1)^2}_{+1}$

c. aux.

$P\,\dfrac{1}{x^2+2x+2} = P\,\dfrac{1'}{(x+1)^2+1} = \text{arctg}(x+1)+C$

$= \displaystyle\lim_{b \to +\infty} \left[\text{arctg}(x+1)\right]_0^b = \lim_{b \to +\infty} \Big[\text{arctg}(b+1) - \underbrace{\text{arctg}(1)}_{\frac{\pi}{4}}\Big]$

$= \dfrac{\pi}{2} - \dfrac{\pi}{4} = \dfrac{\pi}{4}$ // converge

b) $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad D = \{x \in \mathbb{R} : \sqrt{1-x^2} \neq 0 \wedge 1-x^2 \geq 0\} =]-1,1[$ $\quad$ se fosse fechado não havia problema

$1-x^2 \neq 0 \qquad x^2 \leq 1$
$x^2 \neq 1$
$x \neq 1 \vee x \neq -1$

(2ª espécie)

$\displaystyle\lim_{b \to 1^-} \int_0^b \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{b \to 1^-} \left[\text{arcsen}\,x\right]_0^b = \lim_{b \to 1^-} \Big[\text{arcsen}\,b - \underbrace{\text{arcsen}\,0}_{=0}\Big] = \dfrac{\pi}{2}$ converge

2ª Frequência 11/01/2013 - G. III

2.ii) Calcule, caso existam, os seguintes integrais

$$\int_0^1 \frac{1}{x\sqrt{1-\ln x}}\,dx \qquad D=\{x\in\mathbb{R}: \underbrace{1-\ln x\geq 0}_{1-\ln x\geq 0} \wedge x>0 \wedge \underbrace{x\sqrt{1-\ln x}\neq 0}_{x\neq 0\ \wedge\ 1-\ln x\neq 0}\} =]0;e[$$

(2ª espécie)

$\ln x<1 \Leftrightarrow x<e \wedge x>0$

$$=\lim_{a\to 0^+}\int_a^1 \frac{1}{x\sqrt{1-\ln x}}\,dx$$

c.aux. $P\frac{1}{x\sqrt{1-\ln x}} = -P\underbrace{-\frac{1}{x}}_{u'}\underbrace{(1-\ln x)^{-1/2}}_{u^{\alpha}} = -\frac{(1-\ln x)^{1/2}}{1/2}$

$$=-2\lim_{a\to 0^+}\left[\sqrt{1-\ln x}\right]_a^1 = -2\lim_{a\to 0^+}\Big(\underbrace{\sqrt{1-\ln 1}}_{=1}-\underbrace{\sqrt{1-\ln a}}_{=(+\infty)}\Big) \qquad = -2\sqrt{1-\ln x}+c,\ c\in\mathbb{R}$$

$= -2(-\infty) = +\infty$ diverge

Calcule

a) $\int_e^{+\infty}\frac{1}{x\sqrt{\ln x-1}}\,dx \qquad D=\{x\in\mathbb{R}: x\sqrt{\ln x-1}\neq 0 \wedge \ln x-1\geq 0 \wedge x>0\} =]e;+\infty[$

$x\neq 0 \wedge \ln x\neq 1 \wedge \ln x-1\geq 0 \wedge x>0$

$x\neq 0 \wedge x>e \wedge x>0$

(3ª espécie)

problema de domínio + intervalo (+∞)

$$=\lim_{a\to e^+}\int_a^{\boxed{e^2}}\frac{1}{x\sqrt{\ln x-1}} + \lim_{b\to+\infty}\int_{\boxed{e^2}}^b \frac{1}{x\sqrt{\ln x-1}}\,dx = +\infty \text{ diverge}$$

(2ª espécie) ① + (1ª espécie) ② = (3ª espécie)

escolher qualquer valor dentro do intervalo

c.aux.

$$P\frac{1}{x\sqrt{\ln x-1}} = P\frac{1}{x}(\ln x-1)^{-1/2} = \frac{(\ln x-1)^{1/2}}{1/2}+c = 2\sqrt{\ln x-1}+c,\ c\in\mathbb{R}$$

① $2\lim_{a\to e^+}\left[\sqrt{\ln x-1}\right]_a^{e^2} = 2\lim_{a\to e^+}\left(\sqrt{\ln e^2-1}-\sqrt{\ln a-1}\right) = 2\lim_{a\to e^+}\left(1-\sqrt{\ln a-1}\right) = 2(1-0)=2$ converge

② $2\lim_{b\to+\infty}\left[\sqrt{\ln x-1}\right]_{e^2}^b = 2\lim_{b\to+\infty}\left[\sqrt{\ln b-1}-\sqrt{\ln e^2-1}\right] = 2\lim_{b\to+\infty}\left[\sqrt{\ln b-1}-1\right] = +\infty$ diverge

b) $\int_0^{+\infty}\frac{x-1}{2x^2+x+3}\,dx \qquad D=\{x\in\mathbb{R}: \underbrace{2x^2+x+3\neq 0}_{\text{n tem zeros}}\} = \mathbb{R}$

(1ª espécie)

$$=\lim_{b\to+\infty}\int_0^b \frac{x-1}{2x^2+x+3}$$

c.aux.

$$\frac{1}{4}P\frac{4(x-1)}{2x^2+x+3} = \frac{1}{4}P\frac{1+4x-4-1}{2x^2+x+3} =$$

$$=\frac{1}{4}\left[P\frac{4x+1}{2x^2+x+3} - 5P\frac{1}{2x^2+x+3}\right] =$$

$$=\frac{1}{4}\ln(2x^2+x+3) - \frac{5}{4}P\frac{1}{2\left(x^2+\frac{x}{2}+\frac{3}{2}\right)} =$$

$$=\ldots-\frac{5}{8}P\frac{1}{x^2+\frac{x}{2}+\frac{3}{2}} = \ldots-\frac{5}{8}P\frac{1}{\left(x+\frac{1}{4}\right)^2+\left(\frac{\sqrt{23}}{4}\right)^2} = (*)$$

$$P\frac{u'}{a^2+u^2} = \frac{1}{a}\operatorname{arctg}\left(\frac{u}{a}\right)$$

$$x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2+\frac{3}{2}-\left(\frac{1}{4}\right)^2$$

$$\left(x+\frac{1}{4}\right)^2+\frac{3}{2}-\frac{1}{16} = \underbrace{\left(x+\frac{1}{4}\right)}_{u}^2+\underbrace{\left(\frac{\sqrt{23}}{4}\right)}_{a}^2$$

$$(*) = \ldots-\frac{5}{8}\cdot\frac{4}{\sqrt{23}}\operatorname{arctg}\left(\frac{x+\frac{1}{4}}{\frac{\sqrt{23}}{4}}\right)$$

$$=\ldots-\frac{5}{2\sqrt{23}}\operatorname{arctg}\left(\frac{4x+1}{\sqrt{23}}\right)+c,\ c\in\mathbb{R}$$

Exemplos ① estude o sentido das concavidades da função F definida por:

$$F(x) = \int_x^1 f(t)\,dt \quad \text{em que} \quad f(t) = \int_0^{2t} e^{z^2}(1-z^2)\,dz$$

$$F'(x) = \frac{d}{dx}\left(\int_x^1 f(t)\,dt\right) = 0 - f(x)\cdot x' = -f(x)$$

$$F''(x) = \frac{d}{dx}\left[-f(x)\right] = -\frac{d}{dx}\left[\int_0^{2x} e^{z^2}(1-z^2)\,dz\right] = -e^{(2x)^2}(1-(2x)^2)\cdot(2x)'$$

$$= -2\,e^{4x^2}(1-4x^2)$$

⊕ $-\frac{1}{2}$ $\frac{1}{2}$ vai trocar

		$-1/2$		$1/2$	
F''	+	0	−	0	+
F	∪		∩		∪

② seja $h: \mathbb{R} \to \mathbb{R}$ a função definida por: $h(x) = \int_2^{2+\ln(x^2+1)} (e^{t^2}+t^2)\,dt$ e estude monotonia e extremos:

$$h'(x) = \frac{d}{dx}\left[\int_2^{2+\ln(x^2+1)} (e^{t^2}+t^2)\,dt\right] = \left(e^{[2+\ln(x^2+1)]^2} + [2+\ln(x^2+1)]^2\right)\cdot[2+\ln(x^2+1)]'$$

$$= \underbrace{\left(e^{[2+\ln(x^2+1)]^2} + [2+\ln(x^2+1)]^2\right)}_{\oplus}\cdot\frac{2x}{x^2+1}$$

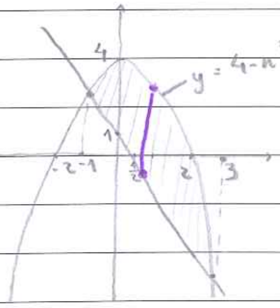
	$-\infty$	0		$+\infty$
h'	−	0	+	
h	↘	m	↗	

mínimo local em $(0; h(0))$ ⟶ $\int_2^2 (\ldots)\,dt = 0$

③ Determine a área da região a) $\begin{cases} y \leq 4-x^2 \\ 2x+y \geq 1 \end{cases}$

c.aux. $4-x^2=0 \Leftrightarrow x=\pm 2$

$y = 1-2x$

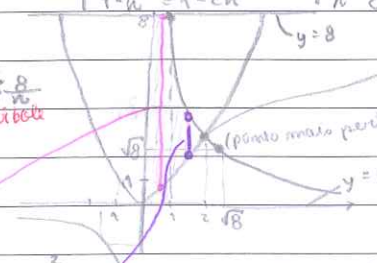


$$A = \int_{-1}^{3} (4-x^2)-(1-2x)\,dx = \int_{-1}^{3}(3-x^2+2x)\,dx = \left(3x - \frac{x^3}{3} + x^2\right) =$$

$$= (9-9+9) - \left(-3+\frac{1}{3}+1\right) = \frac{32}{3}$$

calc. aux $\begin{cases} y = 4-x^2 \\ y = 1-2x \end{cases} \Leftrightarrow \begin{cases} — \\ 4-x^2 = 1-2x \end{cases} \Leftrightarrow \begin{cases} — \\ x^2-2x-3=0 \end{cases} \Leftrightarrow \begin{cases} — \\ x = \frac{2\pm 4}{2} < \begin{matrix}\boxed{3}\\ \boxed{-1}\end{matrix} \end{cases}$

b) $\begin{cases} y \geq x^2 \\ x y \leq 8 \Leftrightarrow y \leq \frac{8}{x} \\ 0 \leq y \leq 8 \\ x \geq 0 \end{cases}$ equação hipérbole

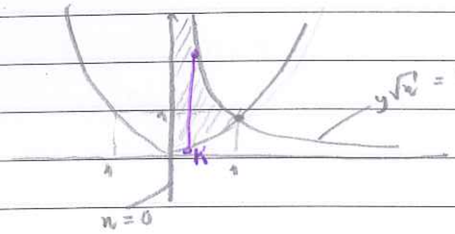


$\begin{cases} x\cdot y = 8 \\ y = x^2 \end{cases} \Leftrightarrow \begin{cases} x^3 = 8 \\ — \end{cases} \Leftrightarrow \begin{cases} x = 2 \end{cases}$

$$A = \int_0^1 (8-x^2)\,dx + \int_1^2 \left(\frac{8}{x} - x^2\right)dx = \left(8x - \frac{x^3}{3}\right)\Big|_0^1 + \left(8\ln|x| - \frac{x^3}{3}\right)\Big|_1^2 = \left(8-\frac{1}{3}\right) - 0 + \left(8\ln 2 - \frac{8}{3}\right) - \left(0 - \frac{1}{3}\right)$$

$$= \frac{16}{3} + 8\ln 2$$

c) $\{(x;y)\in\mathbb{R}^2 : y\sqrt{x}\le 1 \wedge y\ge x^2 \wedge x\ge 0\}$



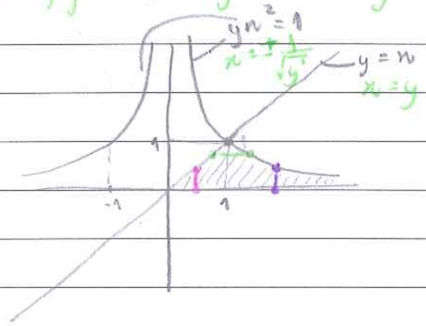
$y\sqrt{x}=1 \Leftrightarrow y=\frac{1}{\sqrt{x}}$

$\begin{cases} y\sqrt{x}=1 \\ y=x^2 \end{cases} \Leftrightarrow \begin{cases} x^2\sqrt{x}=1 \\ \text{—} \end{cases} \Leftrightarrow \begin{cases} x^{5/2}=1 \\ \text{—} \end{cases} \Leftrightarrow \begin{cases} x=1 \\ \text{—} \end{cases}$

$$A=\int_0^1\left(\frac{1}{\sqrt{x}}-x^2\right)dx=\lim_{K\to 0^+}\int_K^1\left(\frac{1}{\sqrt{x}}-x^2\right)dx=\lim_{K\to 0^+}\left(2\sqrt{x}-\frac{x^3}{3}\right)\Big|_K^1=$$

$$=\lim_{K\to 0^+}\left[\left(2-\frac{1}{3}\right)-\left(2\sqrt{K}-\frac{K^3}{3}\right)\right]=\frac{5}{3}$$

d) $\{(x;y)\in\mathbb{R}^2 : yx^2\le 1 \wedge 0\le y\le x\}$



$$A=\int_0^1(x-0)\,dx+\int_1^{+\infty}\left(\frac{1}{x^2}-0\right)dx=$$

$$=\frac{x^2}{2}\Big|_0^1+\lim_{b\to+\infty}\int_1^b\frac{1}{x^2}\,dx=\frac{1}{2}-0+\lim_{b\to+\infty}\left(-\frac{1}{x}\right)\Big|_1^b=$$

$$=\frac{1}{2}+\lim_{b\to+\infty}\left(-\frac{1}{b}+1\right)=\frac{3}{2}$$

forma alternativa

$$A=\int_0^1\left(\frac{1}{\sqrt{y}}-y\right)dy=\lim_{K\to 0^+}\int_K^1\left(\frac{1}{\sqrt{y}}-y\right)dy=\lim_{K\to 0^+}\left(2\sqrt{y}-\frac{y^2}{2}\right)\Big|_K^1=\lim_{K\to 0^+}\left[\left(2-\frac{1}{2}\right)-\left(2\sqrt{K}-\frac{K^2}{2}\right)\right]$$

$$=\frac{3}{2}$$

exercício mini-teste 2010

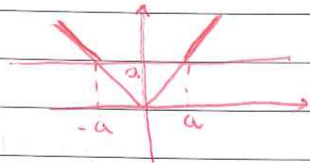
$f(x) = \ln(|x|-2)$

$D_f = \{x \in \mathbb{R} : |x|-2>0\} =]-\infty;-2[\cup]2;+\infty[$

$|x|>2$

$x>2 \vee x<-2$

$\cdot\ |x|>a \Leftrightarrow x>a \vee x<-a$



· intersecção com os eixos

$\rightarrow xx: f(x)=0 \Leftrightarrow \ln(|x|-2)=0 \vee x \in D_f$

$\Leftrightarrow |x|-2 = e^0 \vee x \in D_f$

$\Leftrightarrow |x| = 3 \vee x \in D_f$

$\Leftrightarrow (x=3 \vee x=-3) \vee (x<-2 \vee x>2)$

$\therefore (3;0)\ .\ (-3,0)$

$\rightarrow yy: f(0)$ não existe

$$g(x) = \begin{cases} \frac{\pi}{2} - \operatorname{arcsen}(x) & \text{se } x \leq 1 \\ \sqrt{x-1} & \text{se } x > 1 \end{cases}$$

$D_g = \{x \in \mathbb{R} : (-1 \leq x \leq 1 \wedge x \leq 1) \vee (x-1 \geq 0 \wedge x > 1)\}$

$-1 \leq x \leq 1 \vee x > 1$

$D_g = [-1;+\infty[$

· intersecção com os eixos

$\rightarrow xx: g(x)=0 \Leftrightarrow \left(\frac{\pi}{2} - \operatorname{arcsen} x = 0 \wedge -1 \leq x \leq 1\right) \vee \left(\sqrt{x-1}=0 \wedge x>1\right)$

$\Leftrightarrow \left(\operatorname{arcsen} x = \frac{\pi}{2} \wedge -1 \leq x \leq 1\right) \vee (\underline{x=1 \wedge x>1})$ ✗

$\Leftrightarrow x = \operatorname{sen}\frac{\pi}{2} \wedge -1 \leq x \leq 1$

$\Leftrightarrow x = 1 \wedge -1 \leq x \leq 1$

$(1;0)$

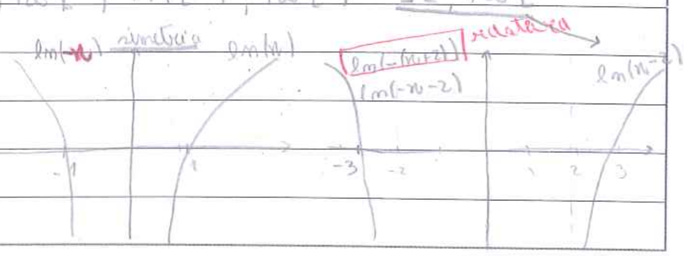
$\rightarrow yy: g(0) = \frac{\pi}{2} - \operatorname{arcsen}(0) = \frac{\pi}{2}$

$\left(0;\frac{\pi}{2}\right)$

· não, pois função não é injetiva, por exemplo, $f(3) = f(-3) = 0$, logo esta função não admite inversa

· $f+g$ $\quad D_{f+g} = D_f \cap D_g = (]-\infty;-2[\cup]2;+\infty[) \cap ([-1;+\infty[) = \underline{]2;+\infty[}$

$$f(x) = \begin{cases} \ln(x-2) & \text{se } x>2 ✓ \\ \ln(-x-2) & \text{se } x<-2 \end{cases}$$



· $(f+g)x = \underline{\ln(x-2)} + \sqrt{x-1}$

	2		$+\infty$
f		$\ln(x-2)$	
g		$\sqrt{x-1}$	
		$\ln(x-2)+\sqrt{x-1}$	

Inversa

1) pela definição $A^{-1} \cdot A = A \cdot A^{-1} = I$

2) pela condensação $[A \,\vdots\, I] \rightarrow [I \,\vdots\, A^{-1}]$

3) pela adjunta $A^{-1} = \frac{1}{|A|} \operatorname{adj}(A) = \frac{1}{|A|} [\operatorname{cof}(A)]^T = \frac{1}{|A|} [(-1)^{i+j} \cdot |A_{ij}|]^T$

recorde que ...

se A e B duas matrizes quadradas de ordem n e α escalar então:

- $|A+B| \neq |A| + |B|$
- $|AB| = |A| \cdot |B| = |B| \cdot |A|$
- $|A^{-1}| = \frac{1}{|A|}$ (se A for regular)
- $|A^T| = |A|$
- $|\alpha A| = \alpha^n \cdot |A|$